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SHORT-TIME CREEP, (U)

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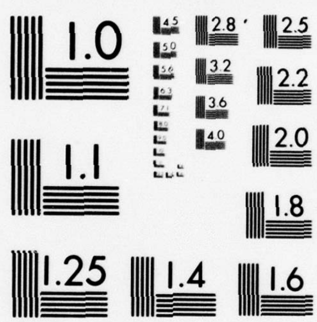

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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

SHORT-TIME CREEP

By

Yu. N. Rabotnov, S. T. Mileyko

FOREIGN TECHNOLOGY DIVISION

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FTD- ID(RS)T-1445-78

UNEDITED MACHINE TRANSLATION

FTD-ID(RS)T-1445-78 19 October 1978

MICROFICHE NR: FTD-78-C-001426

CSC71301301

SHORT-TIME CREEP

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English pages: 344

Source: Kratkovremennaya Polzuchest'Izd-Vo "Nauka," Moscow, 1970, pp. 1-220

Country of Origin: USSR
This document is a machine translation.

Requester: FTD/TQTA
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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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SHORT-TIME CREEP.

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No typing.

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PREFACE.

The authors produced the experimental investigation of short-time creep of a series of structural alloys. The results of this investigation proved to be possible to present in the form, allow/assuming direct use during calculations for the strength of the articles, workers at high temperatures brief time. The first chapters of the book contain the description of the calculation methods

last/latter chapter bears the character of handbook, in it are given the necessary for performance calculations of materials. It is logical that the batch of the tested alloys proved to be limited, and in this sense reference part cannot pretend to completeness. Therefore the authors considered as the possible to give the short description of experimental installations and procedure of obtaining the characteristics of rapid creep.

Under rapid creep in this book, are understood such processes of creep of metals, when the considerable strain, which disturbs the structural/design function of article, is accumulated for relatively short time. The given determination is conditional, since fixation of precise boundary/interfaces for strain and time of service is difficult. From the point of designer, the maximum permissible strain composes the value of order 1-2o/o, it is rarely more, let us say, to 5o/o.

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The range of times in question stretches approximately from 2-3 s. to 1000 s. Since the designer deals usually with allowable stresses, and the dependence of strain on stress during creep is sharply nonlinear and a small change in the stress very strongly affects the amount of strain, precise establishment of boundary/interfaces on strain and on

time dces not have special sense.

The given determinaticn of rapid creep actually characterizes not so much the duraticr cf prccess as range cf stresses and temperatures which subsequently everywhere is specified.

During the calculation of articles, which are fcund under conditions of rapid creep, the designer is lccated in more advantageous position, than during the planning of the objects, calculated to high lifetimes. Cn cre hand, sufficiently complete experimental data for a material can be obtained for entire range of stresses, temperatures and times; therefore drcps off the necessity for the extrapolation of empirical curves, which always introduces some doubts. On the other hand, the law governing rapid creep they are prcved to be sufficiently simple, they can be represented in the form of some equations which directly are utilized for calculations. Recently one of the authors released the monograph "creep of the elements of construction/designs" (Fizmatgiz, Moscow, 1966, subsequently will be called PEK); therefore during the composition of this bcok, the authors tried nct to duplicate/back up/reinforce the material of this monograph. Nevertheless the bcok can be read and independent of PEK.

Here are examined cnly the simplest prcblems of rapid creep,

which are solved with the aid of the elementary procedures of strength of materials.

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The special feature/peculiarity of these problems is the fact that almost all the solutions are obtained as result of numerical count with the use of experimental curves. Some of the described methods are used not only for the problems of rapid creep; according to the theory of steady-state creep or or isochronal curves it is possible to calculate prolonged processes, if there is sufficient initial information. On the other hand, the known and described in the appropriate literature methods of the solution of the problems of the theory of creep are used for the analysis of rapid creep. Therefore such questions as bend of plates, creep of rotary disks, stability during creep and some others, here are not set forth, the reader can find the presentation of the corresponding methods, for example, in EEK.

The presence sufficient to detailed bibliography in the mentioned book allowed the authors not to give here references.

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Chapter I.

GENERAL INFORMATION ABOUT CREEP OF METALS AND OF ALLOYS.

§1. Creep test and curves creep of metals.

Creep is called usually the slow deformation of article under the action of constant load. The property of creep possess plastics, the materials of organic origin and metals. Of metals and alloys, creep is developed only at sufficiently high temperature. Real articles frequently undergo the action of alternating loads which cause instantaneous elastic and plastic strains, and also creep. Creep strain is not determined by the unambiguously applied load, but it depends on the law of a change in the load with time.

The basic information about creep is obtained as a result of standard tensile tests by constant load. Data these tests occur in the form of the so-called curves of creep, which give the dependence of elongation per unit length e on time t. Typical curve of creep is depicted in Fig. 1. Here e_0 - instantaneous deformation, which depends only on the applied stress. On the first section the

deformation rate alternating/variable, it diminishes in the course of time, reaching the minimum value $v(\sigma)$. On the second section the creep rate is constant, $d\epsilon/dt=v(\sigma)$. Finally, by the third section velocity again grow/rises until ensues the rupture of specimen/sample. Not always the process of creep proceeds in accordance with the schematic indicated. To curve can no the first or third section, the first it can immediately transfer/convert into the third. Sometimes is observed the more complex behavior of material.

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A decrease in the velocity of strain on the first section is explained by strengthening, the strain of specimen/sample is accompanied by such structural changes in the metal which impede creep. Output/yield to the second section indicates the depletion of the ability of material to be strengthened. Accelerated creep on the third section can be explained by two reasons. First, if it occurs with constant load, strain is accompanied by reduction in area, therefore, creep occurs with an increase in the stress, and creep rate completely sharply grow/rises with an increase in the stress. In the second place, the decomposition of metal under conditions of the prolonged action of load occurs as a result of crack formation. At the sufficiently high temperature of crack, they are conceived on grain boundaries. The appearance of cracks decreases the effective

cross-sectional area of specimen/sample and increases, thus, actual stress. Rupture occurs, when microscopic cracks are drawn off and form large rupture crack.

In a number of the cases during the aralysis of decomposition, conditions is necessary to consider both change in the area geometric and a change in the effective area as a result of crack formation.

Instantaneous deformation $e_0(\sigma)$ in turn, consists of two parts: by elastic and instantaneous plastic

$$e_0=\frac{\sigma}{E}+g(\sigma). \qquad (1.1)$$

The elastic part of instantaneous deformation is reversible, the plastic part $g(\sigma)$ irreversible, the more correct recording of the formula given above will be following:

$$e_0=\frac{\sigma}{E}+g(\sigma_{max}). \qquad (1.2)$$

Fig. 1. Typical curve of creep.

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Here σ_{max} - the great value of voltage, reached in the process of loading.

The use of formula (1.2) can be explained as follows. Let us assume that the applied voltage is charged in accordance with the graph of Fig. 2. On section OA, the voltage grow/rises, always occurs the plastic deformation whose value is determined at each torque/moment by the effective stress σ. During the computation of strain, one should use formula (1.1). After point A, the voltage first diminishs, then it grow/rises. Thus far $\sigma \leqslant \sigma_A$, the amount of plastic deformation remains constant, equal to $g(\sigma_A)$, therefore in formula (1.2) it is necessary to accept $\sigma_{max} = \sigma_A$. At point B, voltage $\sigma_B = \sigma_A$, on section BC voltage grow/rises, in this case $\sigma > \sigma_B = \sigma_A$, therefore again plastic deformation at each torque/moment is determined by effective stress and again becomes valid equation

(1.1).

It must be noted that relationship/ratio (1.2) is correct if and only if voltage is constant (either elongation or compression). The laws of plastic deformation during sign change of load the more or less are studied only in the region of normal temperatures.

The process of creep with constant stress follows the instantaneous plastic strain, with varying stress, generally speaking, it accompanies it.

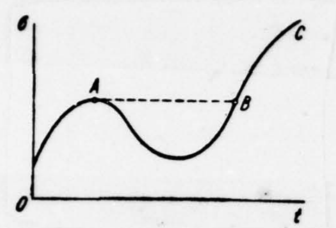

Fig. 2. Example of the program of a change in the stress with time.

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§2. Description of the process of creep.

Creep of metal in essence is the development of the irreversible strain in time. Thus, one should count that creep exists one of the forms of plastic strain. Basic mechanism of creep - slip in the specific atomic planes along determinate directions as a result of the advance of dislocations - the same as and the mechanism of plastic deformation. However, the microscopic picture of creep strain has a number of special feature/peculiarities, which differ it from the microscopic picture of instantaneous plastic strain. Instantaneous plastic deformation, at least in initial stage, is concentrated in the narrow piles of slip planes where the value of local shift/shear is extremely great. The included between these piles regions, which constitute the large part of the volume, remain not deformed. During creep the slip goes more evenly, over a larger

number of the closely spaced atomic planes, moreover the relative displacements of adjacent planes are small - several interatomic distances. This - the plastic deformation, which occurs virtually it is simultaneous by entire volume.

As we will see, during the construction of phenomenological theories, one should differ creep strain from instantaneous plastic strain.

Let us assume in the general case

$$e = p + e_0 = p + \frac{\sigma}{E} + g(\sigma_{max}). \quad (2.1)$$

Here e - full/total/complete strain, p - creep strain. Considering creep as certain form of viscous flow, we must assume that the creep rate at each torque/moment depends on effective stress σ, temperature T and from the structural state of material in given torque/moment. Thus,

$$\dot{p} = v(\sigma, T, q_1, \ldots, q_n). \quad (2.2)$$

Here through q_1, ..., q_n we designated the parameters, characterizing structural state. Thus far we nothing can say about these parameters, and to relationship/ratio (2.2) one should look only as at the symbolic recording of following verbal assertion.

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If as a result of different mechanical and thermal effects specimen/samples of one and the same material are given to the identical structural state (this means that, applying all possible physical methods - metallographic examination, radiography and so forth, we may establish difference in the structure of these specimen/samples), then at identical temperature and with identical voltage creep rate of all specimen/samples will be identical.

It is actually turned out that far not all structural sign/criteria are those determining for creep. In the mechanical theory of creep, are made the hypotheses, which limit a number of structural parameters and expressing them through the values, directly measured in experiment.

Returning to equation (2.1), let us differentiate both of parts of it on time. Then, taking into account (2.2), we obtain

$$\dot{e} = v(\sigma, T, q_1, \ldots, q_n) + \frac{\dot{\sigma}}{E} + \varkappa g'(\sigma)\dot{\sigma}. \quad (2.3)$$

If the process of deformation were begun at torque/moment t=0, and equation (2.3) is related to torque/moment t, then $\varkappa = 1$, if $\sigma(t) > \sigma(\tau)$; $0 \leq \tau < t$, and $\varkappa = 0$, if it is possible to indicate this torque/moment τ, $0 \leq \tau < t$ that $\sigma(\tau) > \sigma(t)$. Equation (2.3) let us apply only if voltage σ retains one and the same sign (either elongation or compression). Apparently, it will be correct and with alternating loadings; however, in this case we do not have enough evidence for

the establishment of form figuring as in it functions.

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§3. Hypothesis of strengthening.

For describing the first sections of curves of creep, they use the hypothesis of strengthening. According to this hypothesis in the right side of formula (2.2) figures as only one structural parameter, the parameter of strengthening. Accepting, that the degree of strengthening grow/rises in proportion to the accumulation of creep strain, we can select as the parameter of strengthening the amount of accumulated creep strain p. Thus,

$$\dot{p} = v(\sigma, T, p). \quad (3.1)$$

Equation (3.1) occasionally referred to as equation of state during creep, although it would be more correctly call it the kinetic equation of creep.

The hypothesis of strengthening underwent repeated experimental check on different materials; it gives the satisfactory description of creep with alternating loads in the first approximation. Some effects by this hypothesis are not described. There are refined versions of the hypothesis of strengthening which make it possible to better reproduce experimental data; however, they lead to more

complex dependences.

The first sections of curves of creep are satisfactorily described by the exponential function of time, so that creep strain is proportional t^m. In conformity and by this the law of hardening is selected in the form

$$\dot{p} = p^{-\alpha} f(\sigma) \qquad \left(\alpha = \frac{1-m}{m}\right). \tag{3.2}$$

If σ=const, then of (3.2) it follows:

$$p = [(1+\alpha) f(\sigma) t]^m.$$

Equation (3.2) does not describe transition from the first phase of creep to the second phase. If we explain transition during the second phase by the depletion of the capability of material for strengthening and not to divide the strain of the steady and transient creep, then it is necessary to assume that

$$\dot{p} = h(p) f(\sigma). \tag{3.3}$$

Function h (p) can be determined, for example, as follows:

$$h(p) = \begin{cases} p^{-\alpha}, & p \leqslant p_c, \\ p_c^{-\alpha}, & p \geqslant p_c. \end{cases}$$

End Section.

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As concerns the selection of the form of the function $f(\sigma)$, then experimental data are satisfactorily approximated by the following expressions:

$$f(\sigma)=B\sigma^{n},$$
$$f(\sigma)=k\exp\frac{\sigma}{A},$$
$$f(\sigma)=2k\,\mathrm{sh}\frac{\sigma}{A}.$$

Here B, n, k, A - constants, which depend on temperature. Constant α is also the function of temperature. Sometimes for the more precise description of real curves of creep, it is necessary to consider index α also voltage-sensitive.

One should especially emphasize that value p, which figures as in the right side of equation (3.1) as the parameter of strengthening, is creep strain, but not full/total/complete plastic deformation. Instantaneous plastic deformation, if its value does not exceed 1-2c/o, does not virtually create strengthening and does not affect creep rate. The considerable plastic deformation of the order of several percentages significantly affects the dependence of creep rate on stress and temperatures, moreover this effect bears fairly

complicated character, there are only separate attempts to consider it in the kinetic equation of creep.

§4. Secondary effects during creep.

Some facts, observed during creep tests, are not explained by the simplest hypothesis of strengthening. So, with the aid of kinetic equation it is not difficult to predict the shape of the curve of creep during a sudden change in the stress. Let for a certain period of time t the stress was σ_1, with $t>t_1$ the stress became equal to σ_2, moreover $\sigma_2>\sigma_1$. Figure 3 shows curve of creep in coordinates p-t and by dashed line - calculated curve, obtained with the aid of equation (3.1). As is evident, immediately after the load application creep rate prove to bes itself above than this is predicted by the hypothesis of strengthening.

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Let us assume now that $\sigma_2<\sigma_1$, the corresponding curve of creep is given to Fig. 4. Immediately after the decrease of stress, creep almost completely stops, and sometimes even it goes in the opposite direction (specimen/sample is shortened). Only during certain time speed becomes such as this follows from the hypothesis of strengthening. Dashed curve here represents the result of calculation

with the aid of equatior (3.1). If is completely unloaded specimen/sample, then it will seem that certain part of creep strain returns in the course of time. Figure 5 gives curve of creep in coordinates e-t, where ϵ - full/total/complete deformation. Cut OA is instantaneous deformation, depending on stress level, it can be either elastic or elasto-plastic. Curve AB - this curve creep with constant stress σ. After unloading the deformation instantly is decreased by value BC, which is elastic deformation.

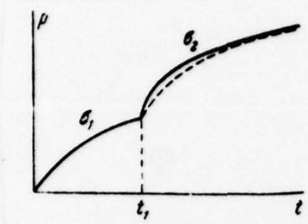

Fig. 3.

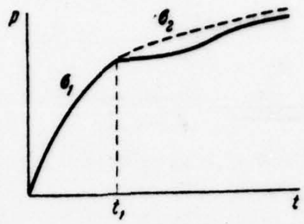

Fig. 4.

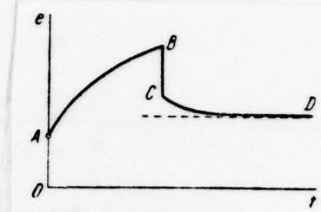

Fig. 5.

Fig. 3. Curve of creep with the gradually increasing stress.

Fig. 4. Curve of creep with gradually diminishing stress.

Fig. 5. Curve of creep and return.

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The unloaded specimen/sample is continued to be reduced; this phenomenon is called return, part CD curved is the curve of return. This curve has horizontal asymptote.

The noted secondary effects do not play the significant role with calculation on creep of the elements of construction/designs. There are more precise theories of creeps, which consider these effects, but the application/appendix of these complex theories for practical target/purposes is difficult.

§5. Instantaneous characteristics of material at high temperature.

Using expression the "instantaneous" deformation, elastic or plastic, we must specify its conditional sense. The instantaneous diagram of deformation we will call the diagram, taken with such speed, at which the effect of which associates deformation creep is unessential. This instantaneous diagram should not be in any way mixed with the dynamic diagram, which is obtained at the very high deformation rates, realized via shock loading.

The modulus of elasticity of material is decreased with an increase in the temperature, the value of the modulus of elasticity can be determined by the static diagram (preferably during unloading, in this case is utilized the phenomenon of the delay of creep during the decrease of the stress, noted into §4). The more convenient method of determining the modulus of elasticity - dynamic, from the measurement of the natural frequencies of the elastic vibrations of specimen/sample or on rate measurement of the passage of the elastic wave.

For the construction of the diagrams of instantaneous deformation at high temperature, is recommended following procedure. Is conducted test series for elongation with the constant velocity of a change in the stress $\dot{s}$=s=const. Figures 6 schematically shows such

curves for the different values $s: s_1 > s_2 > s_3 > \ldots$. Let us examine one of these curves, for example appropriate $\dot{\sigma} = s_2$.

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Abscissa AB of certain point this curved is the full/total/complete strain e up to the torque/moment when stress was achieved value σ, it consists of instantaneous elasto-plastic strain AC and creep strain CB. Creep occurred with varying stress $\sigma = s_2 t$. Let us assume, that from independent experiments to creep with constant stress we determined the form of the function $v(\sigma, T, p)$, figuring as in the right side of equation (3.1). Now we must integrate (3.1) on the assumption that $\sigma = s_2 t$, thus is calculated value CB and is located point C, which belongs to the instantaneous diagram of deformation. It is turned out that, machining by similar machining similarly stress-strain curve, taken with the very strongly separating speeds of loading s (to 3-4 orders), we obtain, with an accuracy to the natural individual spread of the properties of specimen/samples, one and the same curve of instantaneous deformation.

It is obvious that the greater the speed of loading, the lesser manifests itself creep and the nearer actually taken empirical curve to that estimated by the curve of instantaneous deformation. If testing temperature is not too high, then stress-strain curve, taken

with such speed, that entire process of loading lasts 1-2 seconds ($\dot{s}=\dot{\sigma}\approx 10$ kgf/mm² s), for the larger part of the materials, it can be accepted as instantaneous curve.

Figures 7 gives the instantaneous curve, determined by a similar method for alloy BT-14 with T=700°C, and also the common machine diagram, taken at constant velocity of one of the captures of machine, the approximately corresponding rates of deformation of specimen/sample 10^{-4} s^{-1}. As is evident, the difference between them is very considerable.

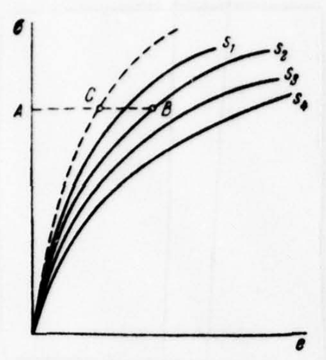

Fig. 6. To the construction of the diagram of instantaneous deformation.

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Tensile tests with the constant velocity of deformation are less convenient for determining of the characteristics of material at high temperature for a number of reasons. First, during the analysis of the work of construction/designs to us there are usually are known the conditions/modes of load, but not the conditions/modes of deformation, therefore, experience/testing material at the fixed/recorded velocities of loading, we can encompass precisely that speed range which interests practice. In the second place, to carry out testing with the constant velocity of the strain (but not with the constant velocity of the motion of capture) it is technically difficult.

For the stress-strain diagrams, taken with the constant velocity of strain, is characteristic the output/yield on the horizontal section, which corresponds to that stress level which would cause the creep rate, equal to the rate of forced deformation (during sufficiently large strain capability for strengthening was exhausted and $\dot{p}=v(\sigma)$).

In §3 was emphasized the absence of the effect of instantaneous plastic deformation on creep rate; however, the opposite effect exists, as a result of the creep of the material, is strengthened with respect to instantaneous plastic deformation. This fact thus far still not found quantitative description, and we it will not take into attention.

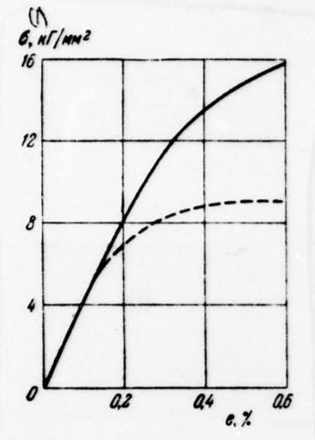

Fig. 7. Curve of instantaneous deformation for alloy BT-14 (T=700°C).

Key: (1). kg/mm².

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The diagram of instantaneous deformation depends substantially on temperature, and relationship/ratio (1.1) must be record/written thus:

$$e_0 = \frac{\sigma}{E(T)} + g(\sigma_{max}, T). \qquad (5.1)$$

We do not avail sufficient experimental data in order to confirm the possibility of applying equation (5.1) in the case of variable temperatures, and are forced to allow this possibility as working hypothesis.

§6. Delayed fracture.

Creep of specimen/sample with sufficient period of loading concludes with destruction. To smaller loads corresponds high duration up to destruction. The decomposition of specimen/sample can have viscous character, i.e., occur during great lengthening and be accompanied by the appearance of a neck, but it can be brittle, i.e., to occur during very small elongation. In the case of brittle decomposition in material, even long before decomposition is observed the appearance of a large number of cracks on grain boundaries. These cracks have the rounded ends, first on boundary/interfaces appear voids or pores, then pores are united into cracks. Cracks increase, as a result of their merging/coalescence, are formed the seen with the naked eye macroscopic cracks, which lead to brittle rupture. One and the same material can fail itself binding with high voltages and, therefore, short times and brittlely with low voltages and long times. Tests for stress-rupture strength consist in the fact that several specimen/samples undergo the action of the loads of different intensity, is record/fixed the time of decomposition. To each value of tensile stress σ corresponds its time of decomposition t_*. The dependence between σ and t_* usually is depicted in logarithmic coordinates, the obtained diagram is called the diagram of stress-rupture strength. The typical diagrams of stress-rupture strength are given schematically to Fig. 8 and 9.

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In the first case at the logarithmic coordinates of point, they lie/rest on one direct/straight, in second diagram consists of two rectilinear cuts. Then part AB corresponds to ductile fracture, part BC - brittle. Not always diagram consists of two straight lines, which have the marked point of intersection, sometimes between the straight lines AB and BC, there is a curvilinear transition region, shown by dashed line, the field of the mixed decomposition. If the diagram of stress-rupture strength takes the form curved, depicted in Fig. 8, this can mean that in experiments is enveloped the range of stresses, which corresponds either to viscous or brittle decomposition. But it can happen, that the straight lines of viscous and brittle decomposition in logarithmic coordinates have identical slope/inclination; then in of direct/straight Fig. 8 there are fields of the viscous, brittle and mixed decompositions. The diagrams of stress-rupture strength can have more complex character, if in material occur phase transformations and if the material of specimen/sample is inhomogeneous (property of surface layer and internal part of the specimen/sample are different).

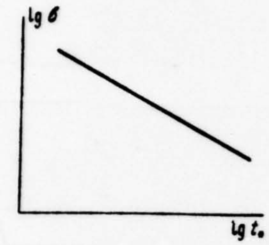

Fig. 8.

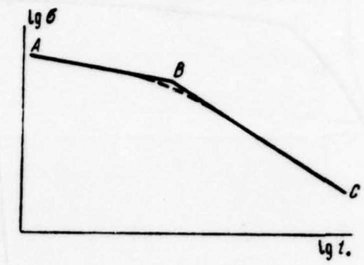

Fig. 9.

Fig. 8. Typical diagram of stress-rupture strength.

Fig. 9. Another typical diagram of stress-rupture strength.

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§7. Ductile fracture.

If decomposition occurs during large strain, it is possible not to consider the section of transient creep and to assume creep rate depending only on stress. In the field of ultimate strains, one should use the logarithmic measure of the strain:

$$e=\ln\frac{x}{x_0}.$$

Here x - length of certain cut, directed along axis of dilatation, x_0 - its initial length.

Disregarding elastic deformation, let us record equation (2.3)

as follows:

$$\dot{e} = g'(\sigma)\,\dot{\sigma} + v(\sigma). \qquad (7.1)$$

Let us consider, that the specimen/sample is dilate/extended by constant force of P. Let us designate through σ_0 the conditional, i.e., that referred to initial cross-sectional area F_0, the stress:

$$\sigma_0 = \frac{P}{F_0}.$$

Then the actual stress at each torque/moment

$$\sigma = \frac{P}{F} = \sigma_0 \frac{F_0}{F}.$$

From the condition of the incompressibility of the material

$$F_0 x_0 = Fx \quad \text{and} \quad \frac{F_0}{F} = \frac{x}{x_0} = \exp(e).$$

Consequently,

$$\sigma = \sigma_0 \exp(e).$$

Differentiating on time, let us find that

$$\dot{\sigma} = \sigma_0 \exp(e)\,\dot{e} = \sigma\dot{e}.$$

Let us eliminate in (7.1) $\dot{e}$, we will obtain

$$\dot{\sigma}\left(\frac{1}{\sigma} - g'(\sigma)\right) = v(\sigma). \qquad (7.2)$$

Let us divide variables and let us integrate. We will obtain

$$t = \int_{\sigma_0}^{\sigma} \frac{\frac{1}{\sigma} - g'(\sigma)}{v(\sigma)}\,d\sigma. \qquad (7.3)$$

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Of real materials the diagram of deformation is usually such, that with certain stress $\sigma = \sigma_*$ the numerator of integrand is turned

into zero. The appropriate value t on formula (7.3) we will designate t_*. This exists time to failure. It is real/actual, if is constructed the graph/diagram of the dependence σ on t on formula (7.3), it will take the form, similar depicted in Fig. 10. The carried out by primes segment of a curve 1 would correspond to the diminishing time, which is impossible. When $t = t_*$ and $\sigma = \sigma_*$ there cannot be equilibria and occurs quick break. Strain up to the torque/moment of achieving time $t = t_*$ exists

$$e_* = g(\sigma_*).$$

This - deformation of uniform elongation. Actually when $e > e_*$ occurs necking, and further analysis, instituted on assumption about uniform elongation, becomes incorrect, the condition of achieving the critical state, which depends on the form of the function g(σ), is establish/installed completely strictly.

A simpler assumption lies in the fact that instantaneous plastic deformation they disregard. Then in formula (7.3) it is necessary to assume g'(σ)=0. For determining the time to failure t'_* one should accept the upper limit of integral the equal ones to infinity, which corresponds to the infinite length of specimen/sample and zero cross-sectional area.

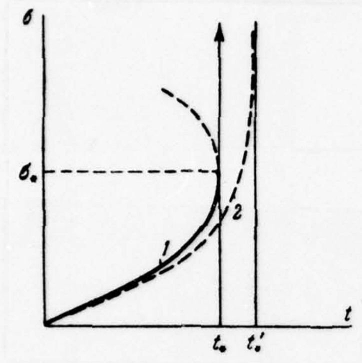

Fig. 10. Graph/diagram of the dependence σ on t on formula (7.3).

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If, for example, is accepted the power law of creep

$$v = A\sigma^n,$$

then is obtained

$$t = \frac{1}{An}\left(\frac{1}{\sigma_0^n} - \frac{1}{\sigma^n}\right) \quad \text{and} \quad t_*' = \frac{1}{An\sigma_0^n}. \qquad (7.4)$$

The dependence between t and σ will be depicted as curved 2, carried out in Fig. 10. This curve has vertical asymptote $t = t_*'$. Value t_*' not very strongly does differ from value t_*, determined taking into account instantaneous plastic deformation.

As concerns the form of the function $g(\sigma)$, it it is possible

satisfactorily to approximate by the exponential function

$$g(\sigma)=a\sigma^m. \quad (7.5)$$

The condition

$$\frac{1}{\sigma_*}-g'(\sigma_*)=0$$

is led to following:

$$am\sigma_*^m=1 \quad (7.6)$$

or

$$e_*=\frac{1}{m}. \quad (7.7)$$

If is disregarded elastic deformation, then curve

$$e=g(\sigma)=a\sigma^m$$

it represents the curve of the instantaneous deformation of material; in this case σ is actual stress, e - logarithmic deformation. In order to obtain the hence conditional diagram of deformation, i.e., the diagram which establish/installs communication/connection between the apparent stress, proportional to the acting force, and logarithmic deformation, we must accept $\sigma=\sigma_0\exp(e)$. Then we obtain

$$e=a\sigma_0^m\exp(me). \quad (7.8)$$

Value σ_0 as function e reaches maximum at certain value of e. In order to establish/install the coordinates of the points of maximum, let us differentiate (7.8) on e. We will obtain

$$1=am\sigma_0^{m-1}\exp(me)\frac{d\sigma_0}{de}+a\sigma_0^m m\exp(me)$$

or

$$m\frac{e}{\sigma_0}\frac{d\sigma_0}{de}=1-me.$$

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The condition of reversal in zero derivative de_0/de is led to condition (7.7); hence it is apparent that e_* is the uniform logarithmic deformation with the short-term rupture of specimen/sample which easily is determined; knowing this value, through formula (7.7) we find index in the law of hardening m.

Under the power law of creep and approximation (7.5) for instantaneous plastic deformation from (7.3) it follows:

$$t_* = \frac{1}{An\sigma_0^n}\left[1 - \frac{mn}{n-m}e_0 + \frac{m}{n-m}(me_0)^{n/m}\right]. \qquad (7.9)$$

It is here accepted

$$e_0 = a\sigma_0^m.$$

If initial stress not is too great, value e_0 differs little from the amount of instantaneous plastic deformation immediately after the load application. This value is low in comparison with unity; therefore, the bracket in formula (7.9) differs little from unity and the application/use of approximation formula (7.4) can be considered justified.

§8. Brittle decomposition.

As already mentioned, the prolonged action of the load of small intensity causes crack formation on grain boundaries. At the

sufficiently high temperatures which will us mainly interest, these cracks have the rounded ends and are formed as a result of merging the pores, which appear on grain boundary. One of the assumed mechanisms of the formation of pores on boundary/interfaces is the diffusion of vacancies. For commercial alloys are essential also the diffusion of impurity atoms to grain boundaries and formation on these boundary/interfaces are brittle phases.

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Being are arrange/located on boundary/interfaces, cracks have diverse orientation; however, predominate cracks or those boundary/interfaces whose planes are nearer anything to the planes, perpendicular to the direction of the action of tensile stress. The presence of cracks attenuate/weakens cross section, decreasing that effective area by which are distributed tensile stresses. Let us designate through ω the degree of the decrease of effective cross-sectional area as a result of cracking. This means that if the geometric cross-sectional area exists F then is the effective area to which it is necessary to relate the acting force, is F(1 - ω). Disregarding a change in the cross-sectional area as a result of creep, we will obtain

$$\sigma = \frac{P}{F(1-\omega)} = \frac{\sigma_0}{1-\omega}. \qquad (8.1)$$

Let us consider that the velocity of crack formation is a function of voltage. This means

$$\dot{\omega} = \varphi(\sigma). \qquad (8.2)$$

Solving (8.1) relatively ω and differentiating, let us find

$$\dot{\omega} = \frac{\sigma_0}{\sigma^2}\dot{\sigma}.$$

Let us introduce in (8.2), let us divide variables and let us integrate, taking into account that with t=0 $\sigma=\sigma_0$, when $t=t_*$ $\omega=1$;

this means that the cracks fill all the section/cut, the remaining effective area vanishes and σ it approaches infinity. We will obtain

$$t_* = \sigma_0 \int_{\sigma_0}^{\infty} \frac{d\sigma}{\sigma^2 \varphi(\sigma)}. \tag{8.3}$$

Formula (8.3) determines the time of brittle destruction. Actually it begins more earlily; at certain sufficiently high value $\sigma = \sigma_*$

occurs instantaneous breakaway, and the upper limit of integral (8.3) it is necessary to take as for equal to σ_*.

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However, we do not avail the data on value σ_* during brittle destruction; on the other hand, the rate of growth in the voltage with an increase in the voltage increases completely rapidly, and the error, which is obtained as a result of replacing upper limit σ_* by infinity, in formula (8.3) is small. Analogous fact was noted into §7 during the analysis of the conditions of ductile fracture.

If is considered that $\varphi(\sigma)$ - the exponential function:

$$\varphi(\sigma) = C\sigma^k,$$

then on formula (8.3) we obtain

$$t_* = \frac{1}{C(1+k)\sigma_0^k}. \tag{8.4}$$

In logarithmic coordinates dependence t_* on σ_0 is depicted as straight line, just as for the case of ductile fracture when is valid formula (7.4). However, generally the slope/inclination of these straight lines different, this is explained the form of the diagram of stress-rupture strength, similar depicted in Fig. 9. It can happen, that n=k; then we will obtain the diagram of Fig. 8.

Of different materials brittle destruction can occur differently. Sometimes cracks are localized near certain section/cut of the specimen/sample, throughout which occurs the destruction. The basic volume of specimen/sample does not contain in this case any noticeable quantity of cracks. It is obvious that in this case creep strain will occur mainly in uncracked volume and creep rate is determined by stress σ_0:

$$v = v(\sigma_0).$$

For other materials and test conditions of crack, are distributed by entire volume more or less evenly; therefore creep rate is determined by actual stress

$$v = v\left(\frac{\sigma_0}{1-\omega}\right).$$

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§9. Mixed destruction.

More common/general/total approach to the problem of destruction lies in the fact that value ω, the degree of cracking, is considered as structural parameter; thus, the process of creep is described by the kinetic equation of creep

$$\dot{e} = v(\sigma, \omega), \tag{9.1}$$

while the process of destruction - by kinetic equation of the destruction

$$\dot{\omega} = \varphi(\sigma, \omega). \tag{9.2}$$

If destruction is accompanied only by insignificant elongation, and voltage σ with constant load can be considered constant, then equation (9.2) is integrated independently, from it it is located $\omega=\omega(\sigma_0, t)$, and is located the torque/moment of destruction as such value $t = t_*$, at which $\omega=1$. After this, substituting in (9.1) $\omega=\omega(t)$ and $\sigma=\sigma_0$, we find the equation of curve of creep e(t). When destruction is accompanied by considerable plastic deformation, it is necessary to consider a change in the cross-sectional area, i.e., to accept $\sigma=\sigma_0\exp(e)$. Now equations (9.1) and (9.2) no longer can be integrated alternately.

In a series of cases, it is turned out that the value of uniform

elongation up to the torque/moment of rupture is constant and does not depend on effective stress. Some authors consider the noted fact general purpose principle. So it will be obtained, if in equations (9.1) and (9.2) right sides are characterized by constant factor. It is real/actual, let us assume v/ϕ=C=const. After dividing (9.1) to (9.2), we will obtain

$$\frac{de}{d\omega}=C.$$

Hence, integrating, we find

$$e=C\omega.$$

When $\omega=1$, $e=e_*$; value e_* is uniform deformation at the moment of destruction. Consequently, $C=e_*$. Now the equations of creep and destruction can be recorded as follows:

$$\dot{e}=v(\sigma,\omega),\quad \dot{\omega}=\frac{1}{e_*}v(\sigma,\omega),\quad \omega=\frac{e}{e_*}.\qquad (9.3)$$

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Simple assumption will here consist in the fact that

$$v=v\left(\frac{\sigma}{1-\omega}\right)\qquad(\sigma=\sigma_0).$$

Let us assume

$$\frac{\sigma}{1-\omega}=z=\frac{\sigma}{1-\frac{e}{e_*}}.\qquad (9.4)$$

If σ=const, then hence follows

$$\dot{e}=e_*\sigma\frac{\dot{z}}{z^2}.\qquad (*)$$

Introducing (*) into the first of equations (9.3) and integrating, let us find

$$t=e_*\sigma\int_{\sigma}^{z}\frac{dz}{z^2v(z)}.\qquad (9.5)$$

This formula gives the equation of curve of creep, since z is expressed as e on formula (9.4). Proposing in (9.5) z=∞ which corresponds $e=e_*$, we will find time to failure t_*.

§10. Linear addition of defectiveness.

Let us accept as the main post of reasonings the diagram of ductile fracture, described into §7. For simplicity we will not take into consideration instantaneous plastic deformation and take the power law of creep. Let us introduce into equation (7.1) $\sigma=\sigma_0\exp(e)$, but let us now consider that σ_0 not it is constant, but it is the assigned/prescribed function from t. Under power law we will obtain

$$\dot{e}=A\sigma_0^n\exp(ne).$$

Let us divide now alternating/variable and let us integrate:

$$\int_0^\infty \exp(-ne)\,de=\int_0^t A\sigma_0^n\,dt.$$

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Integral on the left side is equal to 1/n; thus, the time of destruction is determined from the following condition:

$$\int_0^t nA\sigma_0^n\,dt=1.$$

To this formula it is possible to give another form. If voltage σ_0 was constant, then time to failure t_* would be determined by formula (7.4). It is possible to say that formula (7.4) defines time to failure t_* as function of voltage σ_0, $t_* = t_*(\sigma_0)$. Now destruction condition with alternating loads will be recorded as follows:

$$\int_0^t \frac{dt}{t_*(\sigma)} = 1. \quad (10.1)$$

Dependence $t_*(\sigma)$ is given by the diagram of stress-rupture strength. Now we can forget about the power law, placed as the basis during the derivation of formula (10.1), and use it as follows. Let us assume, that the voltage σ is changed with step/stages in the manner that it is shown on Fig. 11. Let us assume, also that to us is known the diagram of stress-rupture strength for a material at this temperature. This diagram is given to Fig. 12. Logarithmic coordinates are used here only for convenience, it can seem that the points of stress-rupture strength do not lie/rest on one direct, therefore, power dependency in accuracy/precision is not fulfilled, but we nevertheless will use formula (10.1). Let us find from the diagram of stress-rupture strength time $t_*^{(1)}$, for which would occur the destruction with the constantly effective stress σ_1. Let us calculate relation $t_1/t_*^{(1)}$. Let this sense prove to be lesser than unity. Let us find now value $t_*^{(2)}$, which corresponds to voltage σ_2.

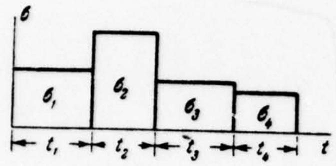

Fig. 11.

Example of the program cf a change in the stress with time.

[Page 31.] Let us calculate sum $t_1/t_*^{(1)}+(t_2-t_1)/t_*^{(2)}$. Let us assume, that this sum is furthermore lesser than one. Continuing process, we will reach finally this step/stage number k, on which is satisfied condition (10.1). Consequently, will be located the value of time t', $t_{k-1}\leqslant t'\leqslant t_k$ such, that

$$\frac{t_1}{t_*^{(1)}}+\frac{t_2-t_1}{t_*^{(2)}}+\frac{t_3-t_2}{t_*^{(3)}}+\ldots+\frac{t'-t_{k-1}}{t_*^{(k)}}=1.$$

This time will be the time of destruction.

Equation (10.1) expresses the so-called principle of the linear addition of defectiveness. It is not difficult to ascertain that equation (10.1) can be obtained, by leaving from the diagram of brittle destruction, described into §8; thus, this principle is not connected with the necessary selection of one or the other initial assumption about the mechanism of destruction. Equation (10.1) can be

considered valid also for the case of variable temperature. The procedure for of calculation of service life with stepped loads, given is above, it is spread also to that case, when with step/stages is changed temperature. For actual calculation it is necessary to have curved long strengths at different temperatures.

During the experimental check of the principle of the linear addition of defectiveness over a wide range of loads and temperatures, are detected some systematic deviations which usually are not so essential not in order to consider them during practical calculations. As a rule, error it is obtained to the side of the understating of service life, i.e., to the side of the increase of safety factor.

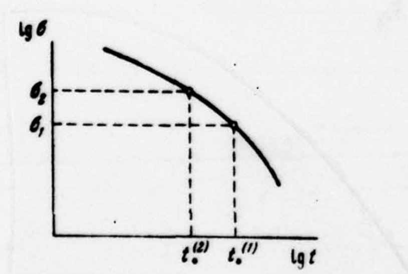

Fig. 12. To the determination of time to failure with gradually changing σ.

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Chapter 11.

RAPID CREEP.

§11. Determination of the working temperature range and stresses.

During solving of practical tasks of calculation for creep and stress-rupture strength, it is necessary to deal with the articles, intended for extremely diverse lifetimes. The assemblies of steam turbines are design/projected taking into account the duration of operation, measured for years and even the decades; however, appear such tasks, when it is required to guarantee the strength of the article, which undergoes the action of load and high temperatures during several minutes and even seconds. In the first case of load and temperature, they must be relatively low, the secondly - it is possible to allow considerable loads at higher temperatures. The behavior of metal these cases will be essentially different, respectively calculated methods for prolonged and rapid creep are proved to be dissimilar.

The sufficiently conditional classification of the fields of

developing the thecry of creep will be follcwing:

1. Prolonged creep, months and years .

2. Creep of average duration - hours and days.

3. Rapid creep - secord and minute.

For the more precision determination of the corresponding ranges, it is convenient tc be turred tc the diagram, depicted in Fig. 13. Along the axis of abscissas, is set aside temperature T, along the axis of ordinates - stress σ. As a result of short-term tensile test, is determined ultimate strength.

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Upper curve is the dependence of tensile strength on temperature. This curve is the upper boundary of the region in which the material generally can be used (with dynamic loads this boundary/interface somewhat is raised; however, these a question here are not examined). Lower dashed curve corresponds to that combination of the temperature and stress, at which is detected creep. This curve can be carried out only completely conditionally. Evidently, physical creep limit does not exist and the determination of the lower boundary of the region

of creep depends on the tolerance accepted for the amount of strain (or the deformation rate).

Let us assume now, that us they interest the cell/elements the lifetime of which is limited by certain maximum time t. As a result of experiments for stress-rupture strength, we can at each temperature determine the stress, which causes destruction for time t, the corresponding curve of stress-rupture strength in coordinates σ - T for the fixed/recorded time is depicted in Fig. 13 (curve $\sigma_{д.п}$).

It is obvious that the calculations for creep make sense when creep strain is not too small in comparison with elastic and instantaneous elasto-plastic strain. So, if creep strain never does exceed 10o/o of instantaneous, scarcely has its sense to consider, it is complete sufficient to perform calculations by the methods of the theory of elasticity and theory of plasticity. For any temperature it is possible to determine conditional creep limit - stress with which creep strain for time t does not exceed the specific portion/fraction of instantaneous deformation (for example, 10o/o).

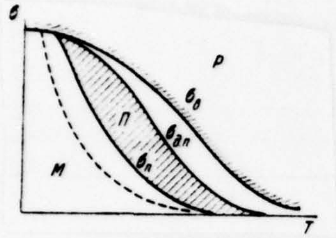

Fig. 13. To the classification of the fields of application of theories of creep.

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Let us note that this determination of conditional creep limit does not coincide with conventional. Let us conduct in plane σ - T dependence curve of conditional creep limit, determined in the sense indicated, from temperature (curve $\sigma_п$). Shaded field $П$, included between curves $\sigma_{д.п}$ and $\sigma_п$, determines that working temperature range and stresses, in which article must be relied on creep. In field M, it suffices to consider only instantaneous deformations. Field P, arrange/located above line, which consists of the segments of the curve of tensile strength and curved stress-rupture strength, is the field of destruction.

The boundary of the region of destruction at low temperatures does not depend on duration t, at high temperatures it depends on t

completely strongly. The position curved σ_n also to a great degree depends on t; as a result it is turned out that field σ - T, in which it is necessary to consider creep, for different service lives of article it is separated strongly.

Figures 14 schematically depicts the temperature ranges and stresses for rapid creep Π_1 and prolonged creep Π_2. Upper boundary of these fields, i.e., the curve of delayed fracture, it is necessary to show, introducing the specific safety factor. Taking into account this fact are constructed fields Π_1 and Π_2, they proved to be those not having common points.

Diagrams of the type of of those depicted in Fig. 13 and 14 actually can be constructed only in rare cases, this it is usually for insufficient experimental data; therefore we were restricted only to diagrams.

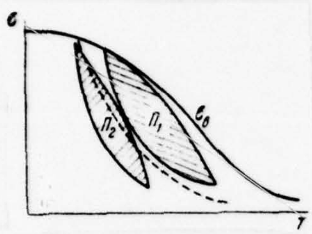

Fig. 14. Temperature ranges and stresses for short-term and prolonged creep.

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Subsequently we will use similar diagrams in order to indicate the field, in which are actually determined the constants and the functions of the law of creep, without full/total/complete confidence in the fact that the boundary/interfaces of the real field of creep are determined sufficiently accurately.

§12. Distinctive special feature/peculiarities of rapid creep.

Rapid creep we will call creep occurring under such conditions, when for the time, which does not exceed 20-30 minutes, is accumulated creep strain, either comparable in value with instantaneous deformation or which exceeds it. For the larger part of the structural materials, such, as chrome-nickel heat-resistant alloys, many steels of ferrite and austenitic class, titanium alloys and aluminum alloys, and also alloys on the basis of niobium and molybdenum in the range of stresses and temperatures of rapid creep, are noted the following special feature/peculiarities:

1. Creep occurs without strengthening. If is left from the law of hardening (3.2), that describes initial creep, then it appears that the index α is decreased with a temperature rise and stress, during certain combination σ and T, it stops $\alpha=0$. Curve $\alpha=0$ is carried out in Fig. 14 by primes, entire/all field Π_1 prove to bes itself above this curved. Typical curve of creep for the alloy EP-202 with T=900°C is given to Fig. 15. The first section on this curved is absent and first creep occurs with the constant velocity

$$\dot{e}=v(\sigma, T).$$

2. Creep rate $v(\sigma, T)$ depends only on stress and temperature and it does not virtually depend on prehistory. This means that specimen/sample can be subjected to plastic strain, given to it to crawl under load, to change temperature, but each time when stress and temperature take one and the same value creep rate it proves to be one and the same.

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The noted conclusion is valid only in the range of small strains, order 1-2o/o; if we impart to specimen/sample greater plastic strain, it it will cause strengthening and creep rate at the same values σ and T will be less than in the undeformed specimen/sample. It is necessary to keep in mind that the increase of temperature to certain limit, which corresponds to rapid softening as a result of, for example, overaging or to recrystallization, causes the irreversible structural changes. This fact also limits that range, in which the made conclusion is valid.

3. Anisotropy of material does not play significant role. It is known that at the moderate temperatures and with stresses the creep rate depends substantially on the texture of material. The specimen/samples, cut out made of one and the same material along the direction of rolled stock and in transverse direction, detect sharply different properties. At high temperatures this difference of properties is smoothed.

4. Decomposition of specimen/samples during rapid creep occurs

with value of uniform elongation which is approximately constant, i.e., it does not depend on stress and temperature. Decomposition precedes the appearance of the third sections in curves of creep (see Fig. 15).

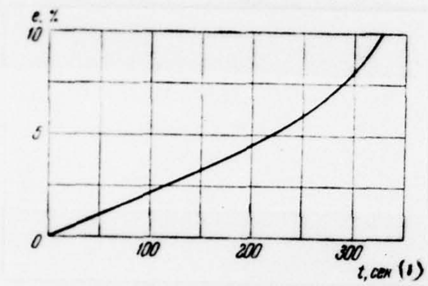

Fig. 15. Typical curve of rapid creep.

Key: (1). s.

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Elementary calculation shows that these third sections cannot be explained by a change in the cross-sectional area, noticeable deviation from straight-line relationship is observed already during the strain of order 1-2c/c. Decomposition occurs as a result of the development of cracks on boundaries of the grains which are drawn off into microcracks, their number can be greatly, in the zone of decomposition, entire/all surface is speckled by cracks.

§13. Equations of rapid creep.

The noted above special feature/peculiarities of the course of the processes of rapid creep make it possible to describe rapid creep by the equation, simpler in comparison with (2.3) and for those being its special case, namely:

$$\dot{e}=\frac{\dot{\sigma}}{E}+\varkappa g'(\sigma)\dot{\sigma}+v(\sigma, T). \qquad (13.1)$$

For the monotonic processes of loading, hence follows:

$$e=\frac{\sigma}{E}+g(\sigma)+\int_0^t v(\sigma, T)\,dt. \qquad (13.2)$$

Value

$$e_0(\sigma) = \frac{\sigma}{E} + g(\sigma)$$

is instantaneous elasto-plastic strain, function $e_0(\sigma)$ is assigned instantaneous stress-strain diagram, understood in that sense, as this is explained in §5.

Determining the characteristics of creep and plasticity of material is manufactured on the basis of the results of the experiments of two types:

1. Creep tests with the constant or changing load. Measuring the tangent inclination in the beginning coordinates to the curves of creep, depicted in Fig. 15, is determined v as function σ and T.

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It must be noted that the straight portions in curves of creep are expressed sufficiently distinctly and value determination of slope/inclination difficulties does not encounter.

2. Tensile tests with constant velocity of loading $\dot{\sigma}$=s. Since function $v(\sigma)$ (at the fixed/recorded temperature) is already known as a result of experiments for creep, creep strain p, which is accumulated in the process of elongation, easily is calculated:

$$p = \int_0^t v(st)\,dt = \frac{1}{s}\int_0^\sigma v(\sigma)\,d\sigma. \qquad (13.3)$$

In order to find the instantanecus defcrmaticn, which corresponds to stress σ, from the full/total/ccmplete strain e, measured according to diagram, they deduct value p, fcund from formula (13.3):

$$e_0(\sigma) = e(\sigma) - \frac{1}{s}\int_0^\sigma v(\sigma)\,d\sigma. \qquad (13.4)$$

As the control of the ccrrectness cf initial assumptions serves the fact that, leaving from stress-strains curve, obtained at the different values of s, we cbtain, with an accuracy to the individual scatter of the properties cf specimen/samples, cne and the same curve $e_0(\sigma)$. This procedure was described in general terms in §5, its realization for the range of the mcderate temperatures encounters the known difficulties, connected with the fact that creep, which accompanies elongation, cccurs mairly cn the first section, i.e., it is acccmpanied by strengthening. The law of hardening during creep is known insufficiently accurately, the simplest hypothesis of strengthening describes real behavior with alternating loads only with known degree of apprcximaticn. In the range of rapid creep, the positicn is proved to be considerably simpler and clearer, the fundamental law, expressed by equation (13.1), is fulfilled with a good degree of accuracy, and on formula (13.4) to it is possible reliably restore/reduce the hypothetical instantaneous curve of

elasto-plastic deformation.

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§14. Formulas for a creep rate.

For practical calculations function $v(\sigma, T)$ it is convenient to assign in certain analytical form. The unavoidable scatter of experimental data makes acceptable ones different analytical approximations of the law of creep; when selecting of these approximations, it is necessary to be guided also by the considerations of convenience in their application/use during calculations. The given here experimental data were machined behind three methods:

1. Exponential dependence

$$\dot{p} = v(\sigma, T) = \varepsilon_e \exp\left(\frac{\sigma}{\sigma_e}\right). \quad (14.1)$$

Here ε_e and σ_e - function of temperature. An exponential dependence of type (14.1) appears in many physical theories of creep. It is unsuitable for low values σ, since with $\sigma=0$ from (14.1) follows different from zero creep rate. In order to correct this deficiency in the exponential law, it they frequently replace by the law of the hyperbolic sine

$$v = 2\varepsilon_e \operatorname{sh} \frac{\sigma}{\sigma_e}. \quad (14.2)$$

Formula (14.2) at high values σ/σ_e gives the results, which do not differ from the results of formula (14.1), with small ones σ creep rate is proved to be linearly voltage-sensitive that it will agree both with the experiment and with physical representations. Designer the range of small stresses, as a rule, does not interest; therefore during calculations it is possible to use formula (14.1), by taking special measures to avoid formal contradictions in the vicinity of that point where σ=0.

2. Power dependence

$$\dot{p} = v(\sigma, T) = \varepsilon_n \left(\frac{\sigma}{\sigma_n}\right)^n. \qquad (14.3)$$

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In formula (14.3) figure as three constants: ε_n, σ_n and n, although independent of them only two. Constant ε_n can be record/fixed on arbitrariness; this certain characteristic rate which is conveniently selected as unity of scale. Values σ_n and ε_n are the functions of temperature. It must be noted that the inversion in zero derivative dv/dσ with σ=0 is physically inadmissible. Analogous fact is encountered in the nonlinear theory of the elasticity: the acceptance of the power law of communication/connection between the stress and the strain leads to the conclusion that the front of the elastic wave is spread with infinite velocity. In some, relatively rare cases during solving of problems of the theory of creep under power law,

appear the formal contradictions whose elimination usually work does not compose.

3. Power dependency with creep limit. Sometimes during processing of experimental data on formula (14.3) of the value of index n, they are proved to be very large (for example, n=20). In these cases it is possible to utilize the following approximation:

$$\left.\begin{aligned} \dot{p} = v(\sigma, T) = \varepsilon_n'\left(\frac{\sigma}{\sigma_n'} - 1\right)^n; \qquad \sigma > \sigma_n', \\ \dot{p} = 0, \qquad \sigma \leqslant \sigma_n'. \end{aligned}\right\} \quad (14.4)$$

Value σ_n' can be the defined limit of creep, although the application/use of formula (14.4) does not completely indicate that is allow/assumed the existence of physical creep limit. In formula (14.4) figure as already three independent constants: $\varepsilon_n', \sigma_n'$ and n, each of which is a function of temperature. The value of index n in formula (14.4) prove to bes itself substantially less than to formula (14.3).

The dependence of the constants of creep on temperature is usually assigned graphically or in the form of tables as this was done in this book. In connection with the exponential law of creep, different authors, being based on one or the other physical theories, obtained the specific form of the dependence of parameters ε_e and σ_e on temperature.

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These theoretical dependences are based on the specific assumptions about the mechanism of creeps which are proved to be valid for pure metals in the specific range of stresses and temperatures. The behavior of commercial alloys is more complicatedly, and for its description it is necessary to use empirical dependence. So, for the conditions of rapid creep were obtained the following empirical formulas:

$$\left.\begin{array}{lll} \varepsilon_e = \varepsilon_{e1} \exp \nu_1 T, & \sigma_e = \sigma_{e1}, & T \leqslant T_*, \\ \varepsilon_e = \varepsilon_{e2} \exp \nu_2 T, & \sigma_e = \dfrac{T_0 - T}{\beta}, & T > T_*. \end{array}\right\} \quad (14.5)$$

Here $\varepsilon_{e1}, \varepsilon_{e2}, \nu_1, \nu_2, \sigma_{e1}, T_0, \beta, T_*$ - constant. These formulas can be useful for interpolation; however, we will use subsequently in essence tables and graphs of the parameters of creep in the function of temperature, only rarely resorting to analytical dependences.

In certain cases, for example for titanium alloys, it is turned out that index n of power law (14.3) is changed comparatively barely in sufficiently wide temperature range. Then it is to convenient perform processing all experimental data at one and the same value n. This is introduced the known error which is partly compensated for by the proper selection of constant σ_n. Then the calculation of the articles, in which the temperature is distributed unevenly, with the constant index n is proved to be substantially simpler.

We did not attempt (with rare exceptions) to select any analytical approximations for a curved instantaneous deformation. During calculations one should utilize directly these curves, systems, if it is necessary, curved deformations for intermediate temperatures by interpolation.

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§15. Third phase of creep and decomposition.

The curve of rapid creep, depicted in Fig. 15, detects the clear expressed third section that it is in any way not reflected in equation (13.1). In order to describe the third section and to predict the torque/moment of decomposition, it is necessary to introduce the parameter of cracking ω, as this was done into §§8 and 9. The fact that the uniform elongation at the moment of rupture e_* is constant, prompts the selection of the kinetic equations of creep and decomposition in the form (9.3). Processing experimental data indicates that the simple assumption, made in §9, namely assumption about **that which**

$$\dot{e} = v\left(\frac{\sigma}{1-\omega}\right), \quad \omega = \frac{e}{e_*}, \qquad (15.1)$$

leads to satisfactory results.

Certain refinement of the analysis, given in §9, will consist in the fact that we will consider in (15.1) a change in the stress with reduction. Let us assume

$$\sigma = \sigma_0 \exp(e) \approx \sigma_0(1 + e).$$

Since $e < e_{**}, e_{*}$ does not exceed 0.06-0.10, this expansion of exponential function gives sufficient accuracy/precision. Now (15.1) it will be rewritten as follows:

$$\dot{e} \approx v\left(\frac{\sigma_0(1+e)}{1-\frac{e}{e_*}}\right) \approx v\left(\frac{\sigma_0}{1-\frac{e}{e_{**}}}\right).$$

Here

$$e_{**} = \frac{e_*}{1+e_*} < e_*.$$

Further integration is conducted just as into §9, and formula (9.5) is replaced by following:

$$t = e_{**}\sigma_0 \int\limits_{\sigma}^{z} \frac{dz}{z^2 v(z)}, \qquad z = \frac{\sigma_0}{1-\frac{e}{e_{**}}}. \qquad (15.2)$$

The torque/moment of decomposition is located through formula (15.2), in which it is necessary to accept $z=\infty$. Value e_* cannot be determined directly with acceptable degree of accuracy; indirectly, on the curves of stress-rupture strength, is located value e_{**}.

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Formula (15.2) in structure does not differ from (9.5), thus, the account of a change of the cross-sectional area in the first approximation, is introduced nothing new in comparison with the calculation, instituted in the diagram of brittle decomposition, is

changed only the mechanical interpretation of the entering the equation constant. Therefore we will designate this constant as before e_{*}, by counting $e_{*}=e_{**}$. Let us use formula (15.2) to the power law of creep. After uncomplicated conversions we will obtain

$$e=e_{*}\left\{1-\left[1-(n+1)\frac{v(\sigma)t}{e_{*}}\right]^{\frac{1}{n+1}}\right\}. \quad (15.3)$$

Here $v(\sigma)$ - the rate of steady-state creep with constant stress σ. This - equation of curve of creep. With small t, by expanding in a series, we will obtain hence

$$e=v(\sigma)t.$$

Decomposition begins when $e=e_{*}$ and the bracket is turned into zero. It is easy to ascertain that in this case stops é=∞. Time to failure is located through the following formula:

$$t_{*}=\frac{e_{*}}{n+1}\frac{1}{v(\sigma)}. \quad (15.4)$$

Of perfecting of the first sections of curves of creep, is located function $v(\sigma)$, i.e., are determined constants σ_n and n. The dependence of time to failure on stress proves to be also exponential with the same index n, which is confirmed sufficiently well on experiment. As a result of processing the curves of stress-rupture strength on formula (15.4) is located constant e_{*}. As the control of the correctness of theory can serve the construction full/total/complete of curve of creep according to equation (15.3) and its comparison with experimental.

If is known value σ_{n}, then time to failure as a result of creep is fulfilled the principle of the linear addition of defectiveness.

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§16. Isochronal curves of creep.

Those noted in §12 special feature/peculiarities of rapid creep are observed not of all materials. Copper and some copper alloys, for example, at any temperatures and with stresses detect the transient creep whose rate is variable. Curves of creep take the form, depicted in Fig. 1. The department/separation of creep from instantaneous plastic strain on that described in §13 diagram for these materials is proved to be difficult.

For the calculation of articles made of similar materials, it is necessary to recommend the method of the so-called isochronal curves of creep, instituted on the hypothesis of aging. According to this hypothesis the full/total/complete strain, which involves instantaneous elasto-plastic strain and creep strain, is the function of stress and time (at the fixed/recorded temperature). Thus,

$$e = f(\sigma, t). \qquad (16.1)$$

The form of this function is assigned by the series of curves of creep (Fig. 16). For convenience in the use, these curves are

reconstructed in coordinates σ - e, in this case, values t serve as the marks of separate curves. A similar rearrancement technique is obvious. Let us draw in Fig. 16 vertical straight line, which corresponds $t=t_1$. Points of intersection with this straight line with curves of creep determine the pairs of values σ and e, value σ exists a mark corresponding of curve of creep, whereas e - ordinate of point of intersection.

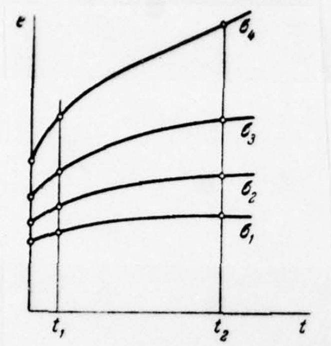

Fig. 16. Series of curves of creep for different ones σ.

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Systems the corresponding points in plane σ - e (Fig. 17), we will obtain curve σ - e for the fixed/recorded time t_1.

The curve of instantaneous deformation belongs, thus, to the family of isochronal curves (upper curve with t=0).

Processing numerous experimental data leads to the following empirical formula with the aid of which can be assign/prescribed entire/all **totality of the** isochronal curves:

$$\sigma = \frac{\varphi(e)}{1 + at^{\beta}}. \qquad (16.2)$$

Fig. 17.

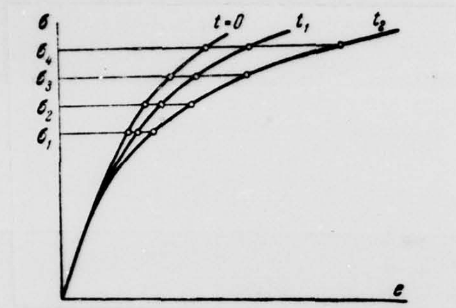

Fig. 18.

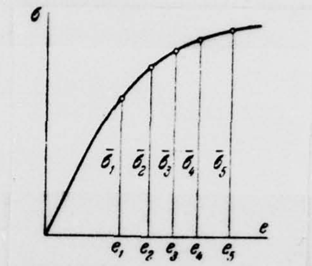

Fig. 19.

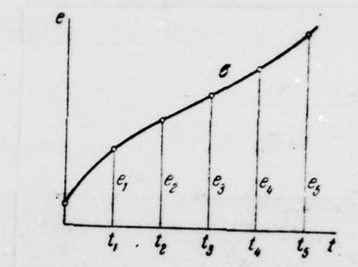

Fig. 17. Tc the construction of iscchronal curves.

Fig. 18. To construction of isochrcnal curves. Instantaneous diagram.

Fig. 19. Tc construction cf isochrcnal curves. Curve of creep.

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Here $\phi(e)$ - the functicn, which establish/installs the dependence between the stress and the strain during irstantaneous deformation, i.e., inversion of a functicn $\epsilon_0(\sigma)$. Formula (16.2) can be rewritten as follows:

$$e(\sigma, t) = e_0\left[\sigma\left(1 + at^{\beta}\right)\right]. \qquad (16.3)$$

Values a and **β are constant at this temperature.** If is used equation (16.2) or (16.3), then the construction of the series of isochronal curves it is reduced to construction of the instantaneous diagram of deformation and definition cf two constants a and β; for this it

suffices to dispose of one curve of creep. It is real/actual, let be given the curve of instantaneous deformation (Fig. 18) and curve of creep with any stress σ (Fig. 19). It is measured in the curve of creep of strain e_1, e_2, e_3, ... for the arbitrary moments of time. Let us plot cuts e_1, e_2, ... along the axis e in Fig. 18 it is measured the corresponding ordinates of the curve of instantaneous deformation, let us designate them $\bar{\sigma}_1$, $\bar{\sigma}_2$, $\bar{\sigma}_3$ .... As a result of (16.3) must be

$$\frac{\bar{\sigma}_k}{\sigma} - 1 = at_k^{\beta}. \qquad (16.4)$$

Let us construct now the graph, depicted in Fig. 20. Along the axis of abscissas, is set aside $\lg t_k$, along the axis of ordinates $\lg\left(\frac{\bar{\sigma}_k}{\sigma} - 1\right)$. Taking the logarithm of formula (16.4), we will obtain

$$\lg\left(\frac{\bar{\sigma}_k}{\sigma} - 1\right) = \lg a + \beta \lg t_k.$$

Small circles on the graph of Fig. 20 must lie/rest on one straight line, the slope/inclination of this straight line exists β, the cut, intercept of ordinates $(t_k = 1)$, is lg a.

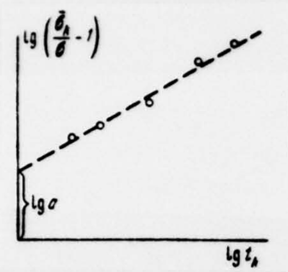

Fig. 20. To the construction of isochronal curves. Dependence $\log\left(\frac{\bar{\sigma}_k}{\sigma}-1\right)$ on $\log t_k$.

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Certainly, more accurate results are obtained, if we take several curves of creep and to transfer points from these curves by the method indicated to the graph of Fig. 20, after which to draw straight line, that passes as close as possible to these points. The noted property of the similarity of isochronal curves, expressed by formula (16.3), makes it possible to fulfill their approximate construction. If for a material is a sufficiently dense grid of curves of creep, it is better to construct isochronal curves directly.

§17. Calculated tasks of the theory of creep.

During creep tests, especially short-term, is detected the large scatter of experimental data. On one hand, this is connected with the natural scatter of the properties of individual specimen/samples, with another - by extremely strong dependence of creep rate on stress and temperature. The uncontrollable deviations of load and temperature from the nominal values, accepted during processing, cause the considerable difference in creep rates. In §14 it was noted, that during processing of experimental data according to the power law of the dependence of velocity on stress with the high ones of the value of index n they are proved to be large, they, as a rule, substantially exceed those values which are obtained for the same material with smaller stresses. But if, for example, n=10, then deviation in load on 2o/o causes deviation in rate of 20o/o. The effect of the error in the determination of temperature can be even more powerful. Tests technique for rapid creep is such, that the accuracy/precision of the fixing of load and temperature unavoidably prove to bes itself somewhat smaller than during tests large and for average duration. Therefore to calculations to creep, one ought not to impose the high requirements of accuracy/precision, since the discussion deals with finding of strains or time to failure.

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The insufficient definition of initial data makes with in practice

useless tendency toward improvement and refinement both of the initial laws governing creep and selective design diagrams, the use of simpler methods here preferably. However, if is is posed problem as follows: it is required to find those values of loads and temperatures, by which during this time will be accumulated specific fixed/recorded creep strain, or: it is required to find load and the temperature, by which the decomposition will occur during preset time, then answer/response becomes more determined and the accuracy/precision of the predictions of the theory is proved to be comparable with that degree of accuracy which usually is presented to engineering calculations. Actually designer always deals with loads, and safety factor is introduced with respect to loads. As a result of the very powerful nonlinear dependence of velocity on load, the introduction of already insignificant safety factor completely guarantees from the accumulation of undesirably large strains.

During the design of the articles, workers in high-temperature range, it is necessary to worry about fulfilling of the following requirements:

a) for the time of service t must not occur the decomposition of article (strength calculation);

b) for the time of service t of the displacement of some points

must not exceed the specific limit, after which is disturbed the structural/design function of article (calculation for rigidity).

Furthermore, in practice are encountered other requirements for providing which it is necessary to perform calculations for creep, for example:

c) the effort/force, which appears between two parts, connected with interference, must not fall lower than specific limit (calculation for relaxation).

Depending on the character of task and on those conditions in which works the article, is necessary to accept one or another method calculation, introducing into the formulation of task and into initial equations these or other simplifications.

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Strength calculation.

1. Total calculation for strength of article must consist in the fact that for each cell/element are comprised kinetic equations of creep and decomposition of type (15.1). As a result of the solution of these equations in principle, it is proved to be possible to find

the apparent time of decomposition, i.e., the separation of article on part. The actual execution of similar calculations is difficult, it will be illustrated based on some examples of model character.

2. Strenhth calculation according to simplified diagram lies in the fact that searches for stress distribution with the aid of equations of creep of type (13.1) without taking into account of effect of parameter ω. The value of the greatest stress is compared from curved stress-rupture strength or is substituted in formula (15.4), from it is located the time to failure. In statically indeterminable systems of stress, they will be redistributed in the course of time, with t=0 these stresses are determined by instantaneous elasto-plastic properties, with $t=\infty$ stress distribution approaches certain stationary distribution which can be obtained from equations (13.1), after reject/throwing in them the terms, which correspond to instantaneous deformation. During the estimation of time to failure from the action of varying stress, one should apply the principle of the addition of defectiveness. Numerous calculations convince of the fact that the account of the redistribution of stresses during transition from initial state to the state of steady-state creep does not introduce essential refinement into the estimation of service life. Calculation according to the stage of steady-state creep gives the lower estimation of service life during creep, i.e., error goes into safety factor.

3. Upper estimation of service life can be found as a result of calculation according to limiting condition - this stressed state, which satisfies equation of equilibrium, during which decomposition of body occurs simultaneously on all points of critical section/cut.

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Calculation for rigidity.

Depending on which value has the maximum strain, which is considered permissible, calculation for rigidity is manufactured according to the diagrams of different degree of accuracy.

1. Total calculation for rigidity. As the basis is set/assumed equation (13.1), instantaneous deformation is considered together with creep strain. Such calculations can be carried out only by numerical methods. The application/use of some approximate procedures makes it possible to sometimes simplify solving of task.

2. Calculation according to steady-state creep. The idea of this approximation method lies in the fact that first is determined the instantaneous deformation of article, and then to it is connected

occurring with constant velocity creep strain. The latter is calculated in the manner that if stress distribution immediately became the fact, which corresponds to steady state, the effect of the process of the redistribution of stresses on the course of creep strain they disregard. If creep strain significantly exceeds instantaneous, error from this assumption is small.

3. Calculation for bulge. During the investigation of rods and thin-walled elements of construction/designs, appear the peculiar tasks of calculation for rigidity. The initial imperfections of geometric form have a tendency to increase under the action of loads; beginning with certain time the rate of growth in the sagging/deflections, caused by initial imperfections, it can become catastrophical large. The tasks of this type occasionally referred to as the tasks of stability, although in this case the application/use of this term not completely is justifiably/legitimate.

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§18. Creep and delayed fracture in the complex stressed state.

Let us examine that case when as the coordinate axes are accepted the principal axle/axes of stresses. The values of principal stresses are designated σ_1, σ_2, and σ_3, the corresponding strains -

e_1, e_2, e_3. There are several variations of the theory of creep in the complex stressed state, the results of their application/use give close results, and when selecting of one or the other version they are frequently guided by the considerations of convenience. Overall writing of the equations of creep can be accepted by following. Let us introduce the equivalent stress S, which is the uniform function of the first degree (it is not compulsory linear) σ_1, σ_2, σ_3. Let us determine this function in such a way that if $\sigma_2=\sigma_3=0$, then

$$S(\sigma_1, 0, 0) = \sigma_1.$$

Now the law of creep taking into account instantaneous plastic strain is record/written as follows:

$$\left.\begin{aligned} \dot{e}_1 &= \frac{1}{E}\left[\dot{\sigma}_1 - \nu(\dot{\sigma}_2 + \dot{\sigma}_3)\right] + \varkappa g'(S)\frac{\partial S}{\partial \sigma_1}\dot{S} + v(S)\frac{\partial S}{\partial \sigma_1}, \\ \dot{e}_2 &= \frac{1}{E}\left[\dot{\sigma}_2 - \nu(\dot{\sigma}_3 + \dot{\sigma}_1)\right] + \left[\varkappa g'(S)\dot{S} + v(S)\right]\frac{\partial S}{\partial \sigma_2}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \quad (18.1)$$

Here E and ν - modulus of elasticity and the Poisson ratio of material of this temperature, g(S) and v(S) - the functions, figuring as in equation (13.1) and determined from experiment to unidirectional tension. Factor $\varkappa = 1$, if $\dot{S}>0$, and $\varkappa = 0$, if $\dot{S}<0$. Let us examine now two most commonly used variations of the theory:

1. Quadratic criterion of creep. Let us assume

$$2S^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2. \quad (18.2)$$

The determined thus value S is called the intensity of stresses σ_0.

Differentiating on σ, let us find

$$\frac{\partial S}{\partial \sigma_1}=\frac{1}{S}\left[\sigma_1-\frac{1}{2}(\sigma_2+\sigma_3)\right].$$

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Formulas (18.1) will be rewritten now thus:

$$\left.\begin{aligned}\dot e_1&=\frac{1}{E}\left[\dot\sigma_1-\nu(\dot\sigma_2+\dot\sigma_3)\right]+\\&\quad+\frac{\varkappa g'(\sigma_0)\dot\sigma_0+v(\sigma_0)}{\sigma_0}\left[\sigma_1-\frac{1}{2}(\sigma_2+\sigma_3)\right],\\\dot e_2&=\frac{1}{E}\left[\dot\sigma_2-\nu(\dot\sigma_3+\dot\sigma_1)\right]+\\&\quad+\frac{\varkappa g'(\sigma_0)\dot\sigma_0+v(\sigma_0)}{\sigma_0}\left[\sigma_2-\frac{1}{2}(\sigma_3+\sigma_1)\right],\\&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}\right\}\qquad(18.3)$$

2. Criterion of greatest shearing stress. Let us assume for certainty $\sigma_1>\sigma_2>\sigma_3$. **Then the given** stress S is determined as follows:

$$S=\sigma_1-\sigma_3.$$

Hence

$$\frac{\partial S}{\partial \sigma_1}=1,\quad \frac{\partial S}{\partial \sigma_2}=0,\quad \frac{\partial S}{\partial \sigma_3}=-1,$$

and formulas (18.1) take the following form:

$$\left.\begin{aligned}\dot e_1&=\frac{1}{E}\left[\dot\sigma_1-\nu(\dot\sigma_2+\dot\sigma_3)\right]+\\&\quad+\varkappa g'(\sigma_1-\sigma_3)\cdot(\dot\sigma_1-\dot\sigma_3)+v(\sigma_1-\sigma_3),\\\dot e_2&=\frac{1}{E}\left[\dot\sigma_2-\nu(\dot\sigma_3+\dot\sigma_1)\right],\\\dot e_3&=\frac{1}{E}\left[\dot\sigma_3-\nu(\dot\sigma_1+\dot\sigma_2)\right]-\\&\quad-\varkappa g'(\sigma_1-\sigma_3)\cdot(\dot\sigma_1-\dot\sigma_3)-v(\sigma_1-\sigma_3).\end{aligned}\right\}\qquad(18.4)$$

Special examination requires the case when two principal stress are

equal to each other.

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Satisfaction of this condition in finite domain sets extremely severe limitation on the form of the possible stressed states, and for solving the task of the theory of creep we are forced to allow known freedom in the selection of the possible distributions of the rates. This freedom follows automatically from equations (18.1). For example, $\sigma_1=\sigma_2>\sigma_3$. Let us assume

$$S=\lambda(\sigma_1-\sigma_3)+(1-\lambda)(\sigma_2-\sigma_3).$$

With $\sigma_1=\sigma_2$ we obtain the previous determination $S=\sigma_1-\sigma_3$, whatever was λ. Hence

$$\frac{\partial S}{\partial\sigma_1}=\lambda,\quad \frac{\partial S}{\partial\sigma_2}=1-\lambda,\quad \frac{\partial S}{\partial\sigma_3}=-1.$$

The law of creep will be recorded now as follows:

$$\left.\begin{aligned}
\dot{e}_1&=\frac{1}{E}[(1-\nu)\dot{\sigma}_1-\nu\dot{\sigma}_3]+\\
&\quad+\lambda[\varkappa g'(\sigma_1-\sigma_3)\cdot(\dot{\sigma}_1-\dot{\sigma}_{,})+v(\sigma_1-\sigma_3)],\\
\dot{e}_2&=\frac{1}{E}[(1-\nu)\dot{\sigma}_1-\nu\dot{\sigma}_3]+\\
&\quad+(1-\lambda)[\varkappa g'(\sigma_1-\sigma_3)\cdot(\dot{\sigma}_1-\dot{\sigma}_3)+v(\sigma_1-\sigma_3)],\\
\dot{e}_3&=\frac{1}{E}[\dot{\sigma}_3-2\nu\dot{\sigma}_1]-\\
&\quad-[\varkappa g'(\sigma_1-\sigma_3)\cdot(\dot{\sigma}_1-\dot{\sigma}_3)+v(\sigma_1-\sigma_3)].
\end{aligned}\right\}\quad(18.5)$$

Here the parameter λ remains not defined, sole limitation lies in the fact that $0\leqslant\lambda\leqslant1$. This arbitrariness usually is removed during solving

cf specific problems, kinematic limitations make it possible to determine the parameter λ in some individual cases. Sometimes, however, indeterminancy/uncertainty remains, and of this consists a known deficiency in the criterion of greatest shearing stress.

During calculations for creep according to isochronal curves, they usually use the equations of the deformation theory of plasticity, replacing the diagram of the plasticity isochronal of curve of creep. With fixed/recorded t the equation this curved is record/written in the form

$$e=e(\sigma).$$

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Let us consider that the same relationship/ratio exists between the intensity of strains and stress intensity. Then the equations of creep take the form

$$\left.\begin{aligned} e_1 &= \frac{e(\sigma_o)}{\sigma_o}\left[\sigma_1-\frac{1}{2}(\sigma_2+\sigma_3)\right], \\ e_2 &= \frac{e(\sigma_o)}{\sigma_o}\left[\sigma_2-\frac{1}{2}(\sigma_3+\sigma_1)\right], \\ e_3 &= \frac{e(\sigma_o)}{\sigma_o}\left[\sigma_3-\frac{1}{2}(\sigma_1+\sigma_2)\right]. \end{aligned}\right\} \qquad (18.6)$$

The equivalent stress for estimating the time of decomposition during creep under conditions of the complex stressed state usually is accepted in the form

$$\sigma_э=\beta\sigma_1+(1-\beta)\sigma_o, \qquad (18.7)$$

where σ_o and σ_1 - intensity of stresses and maximum normal stress respectively, $0 \leqslant \beta \leqslant 1$. Expression (18.7) is the empirical criterion of stress-rupture strength. In certain cases it is possible to use the criterion of maximum normal stress $\sigma_э=\sigma_1$. This is correct, for example, in the case of planar-stressed state with positive principal stresses, in this case, σ_0 insignificantly it differs from σ_1. However, usually the value of value β is close to 0.5; therefore we will accept the criterion of stress-rupture strength in the form

$$\sigma_э = \frac{1}{2}(\sigma_1 + \sigma_o). \qquad (18.8)$$

If is known value $\sigma_э$, then time to failure can be determined by the curved stress-rupture strength, obtained as a result of tensile tests. This is related, strictly speaking, only to brittle decomposition, the time of ductile fracture not at all it can be determined with the aid of local, i.e., comprised for certain fixed/recorded point, the condition of strength. It is necessary to bear in mind, that in the complex stressed state the relation of greatest normal stress, which determines decomposition condition, to octahedral or greatest shearing stress, which determines creep rate, can be substantially greater than during experiment by elongation.

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Hence it follows that the material, which detects with elongation the decomposition of purely viscous character, can give the picture of

brittle decomposition under conditions of the complex stressed state. Therefore stress concentrators can prove to be dangerous for creep conditions even when creep is short-term.

§19. Creep under the action of normal and shearing stress.

We will not here examine the common/general/total equations of creep in the arbitrary complex stressed state, we will be restricted to the simplest and frequently encountered in appendices case when on one of the faces of the cell/element, depicted in Fig. 21, act normal stresses $\sigma_1=\sigma$ and shearing stresses $\tau_{12}=\tau$, whereas remaining faces are free from normal stresses.

Equations for this case are record/written analogously with equations (18.1), namely:

$$\left.\begin{aligned} \dot{e} &= \frac{\dot{\sigma}}{E} + [\varkappa g'(S)\cdot\dot{S} + v(S)]\frac{\partial S}{\partial\sigma}, \\ \dot{\gamma} &= \frac{\dot{\tau}}{G} + 2[\varkappa g'(S)\cdot\dot{S} + v(S)]\frac{\partial S}{\partial\tau}. \end{aligned}\right\} \quad (19.1)$$

Here e - elongation in the direction of effective stress, γ - angle of displacement, S - uniform function of the first degree relative to stress components, is equal to σ with $\tau=0$.

With the quadratic criterion of creep

$$S^2 = \sigma^2 + 3\tau^2 = \sigma^2 + \frac{3}{2}(\tau_{12}^2 + \tau_{21}^2). \quad (19.2)$$

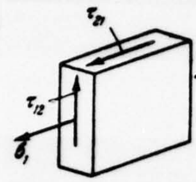

Fig. 21. Diagram of cell/element, which is located under conditions of planar-stressed state.

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Consequently,

$$\left.\begin{aligned}\dot{e} &= \frac{\dot{\sigma}}{E} + [\varkappa g'(S)\cdot\dot{S} + v(S)]\,\frac{\sigma}{S},\\ \dot{\gamma} &= \frac{\dot{\tau}}{G} + 3\,[\varkappa g'(S)\cdot\dot{S} + v(S)]\,\frac{\tau}{S}.\end{aligned}\right\} \quad (19.3)$$

With the criterion of greatest shearing stress

$$S^2 = \sigma^2 + 4\tau^2 = \sigma^2 + 2(\tau_{12}^2 + \tau_{21}^2). \quad (19.4)$$

Consequently,

$$\left.\begin{aligned}\dot{e} &= \frac{\dot{\sigma}}{E} + [\varkappa g'(S)\cdot\dot{S} + v(S)]\,\frac{\sigma}{S},\\ \dot{\gamma} &= \frac{\dot{\tau}}{G} + 4\,[\varkappa g'(S)\cdot\dot{S} + v(S)]\,\frac{\tau}{S}.\end{aligned}\right\} \quad (19.5)$$

§20. Creep during compression.

The large part of the materials during compression detects the same dependence of creep rate on stress, that also with elongation. If is are counted, as usual, tensile strains positive and to compressive strain negative, then equation (13.1) will automatically give correct result, also, in the case of compression when are satisfied the following conditions:

1. Function $g'(\sigma)$ is even:

$$g'(\sigma)=g'(-\sigma).$$

2. Function $v(\sigma)$ is odd:

$$v(\sigma)=-v(-\sigma).$$

As concerns the first condition, applied on function $g'(\sigma)$, during the graphic assignment to function $g(\sigma)$ it always can be carried out. However, the second condition is satisfied only for dependence (14.2); formulas (14.1) and (14.3) this condition do not satisfy.

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Therefore the more correct recording of the dependence of creep rate on stress will be following:

$$\dot{p}=\varepsilon_e \exp\left(\frac{|\sigma|}{\sigma_e}\right)\frac{\sigma}{|\sigma|} \qquad (20.1)$$

instead of (14.1) and

$$\dot{p}=\varepsilon_n \left|\frac{\sigma}{\sigma_n}\right|^{n-1}\frac{\sigma}{\sigma_n} \qquad (20.2)$$

instead of (14.3).

With alternating loads it is necessary to keep in mind that the unique dependence of velocity from stress it is retained only, thus far stress does not reverse the sign, remaining either stretching or compressive. Curve of creep for the case when stress is first positive, and then is reversed the sign, retaining absolute value, it is shown schematically on Fig. 22. The experimental data, which

relate to similar tests, thus far are still insufficient for the construction of theory, and the tasks of creep with alternating stress here be examined will not be. Sometimes for us all the same it is necessary to take into consideration reversal of stress. So, with the curvature of curved profile neutral axle/axis is moved, therefore, at one and the same point the stress reverses the sign. A precise explanation of laws governing creep in the range of small stresses near neutral axle/axis is virtually useless, these stresses by themselves us do not interest, but their contribution to the value of the bending moment is insignificant. Therefore we will consider in such cases of equation (20.1) and (20.2) as formally the valid also for alternating stress.

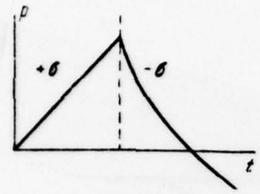

Fig. 22. Curve of creep during reversal of stress.

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Of some materials, for example magnesium alloys, the characteristics of creep during elongation and compression, are essentially different. This means that depending on sign σ in equations (20.1) and (20.2) it is necessary to take the different values of constants.

§21. Potential of creep. Castigliano theorem.

Let us assume

$$\left.\begin{aligned} U(\sigma) &= \frac{\sigma^2}{2E} + \int_0^\sigma g(\sigma)\,d\sigma, \\ \Phi(\sigma) &= \int_0^\sigma v(\sigma, T)\,d\sigma. \end{aligned}\right\} \quad (21.1)$$

Function $U(\sigma)$ is called the density of additional work, function $\Phi(\sigma)$ is called the density of the potential of creep. Equation

(13.1), for the monotonic process when $\dot{\sigma}>0$ and $\varkappa=1$, can be rewritten as follows:

$$\dot{e}=\frac{\partial}{\partial\sigma}\left(\frac{dU}{dt}+\Phi\right). \quad (21.2)$$

Let us examine the now statically determinable farm/truss, which consists of n of rods, the length of rod number i exists l_i, cross-sectional area F_i. To knots of a truss, is applied m of concentrated forces Q_s. Let us determine additional work and the potential of creep for a farm/truss as a whole by the following expressions:

$$\tilde{U}=\sum_{i=1}^{n}U(\sigma_i)F_il_i, \quad \tilde{\Phi}=\sum_{i=1}^{n}\Phi(\sigma_i)F_il_i.$$

In view of the fact that the farm/truss statically determinable, from the equations of statics, we can find the stresses in rods σ_i; these stresses will be the linear functions of external forces Q_s. Therefore $\tilde{U}$ and $\tilde{\Phi}$ are the completely specific functions from the external forces

$$\tilde{U}=\tilde{U}(Q_s), \quad \tilde{\Phi}=\tilde{\Phi}(Q_s) \quad (s=1,\ 2,\ \ldots,\ m).$$

Let us give to external forces arbitrary increases δQ_s, then the stresses in rods obtain completely specific increases $\delta\sigma_i$.

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The system of external forces δQ_s and the system of internal forces $\delta\sigma_i$ will be balanced. Let us record the appropriate condition of

equilibrium, after using the beginning of virtual displacements. Let us accept for virtual displacements the actual velocities of the points of the application/appendix of concentrated forces $\dot{q}_s$ and of the rate of deformation of rods $\dot{e}_i$. We will obtain

$$\sum_{s=1}^{m} \delta Q_s \cdot \dot{q}_s = \sum_{i=1}^{n} \delta\sigma_i \cdot \dot{e}_i l_i F_i. \qquad (21.3)$$

We convert the sum, which stands in right side, with the aid of (21.2),

$$\sum l_i F_i \dot{e}_i \delta\sigma_i = \sum l_i F_i \frac{\partial}{\partial\sigma_i}\left(\frac{dU(\sigma_i)}{dt} + \Phi(\sigma_i)\right)\delta\sigma_i =$$
$$= \sum l_i F_i \delta\left(\frac{dU(\sigma_i)}{dt} + \Phi(\sigma_i)\right) = \delta\left(\frac{d\tilde{U}}{dt} + \tilde{\Phi}\right).$$

Since $\tilde{U}$ and $\tilde{\Phi}$ - function from Q_s, then

$$\delta\left(\frac{d\tilde{U}}{dt} + \tilde{\Phi}\right) = \sum_{s=1}^{m} \frac{\partial}{\partial Q_s}\left(\frac{d\tilde{U}}{dt} + \tilde{\Phi}\right)\delta Q_s.$$

As a result equation (21.3) will be recorded as follows:

$$\sum_{s=1}^{m} \dot{q}_s \delta Q_s = \sum_{s=1}^{m} \frac{\partial}{\partial Q_s}\left(\frac{d\tilde{U}}{dt} + \tilde{\Phi}\right)\delta Q_s.$$

Variations in external forces δQ_s can be assigned on arbitrariness; therefore from the equality written above it follows:

$$\dot{q}_s = \frac{\partial}{\partial Q_s}\left(\frac{d\tilde{U}}{dt} + \tilde{\Phi}\right). \qquad (21.4)$$

Formula (21.4) is the analog of Castigliano's known formula. If there is no creep, then of it it follows:

$$q_s = \frac{\partial \tilde{U}}{\partial Q_s}.$$

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This - Castiglianos's commcn formula for a nonlinearly elastic or elasto-plastic system. If it is possible to disregard instantaneous deformation, then

$$\dot{q}_s = \frac{\partial \tilde{\Phi}}{\partial Q_s}.$$

The Castigliano theorem remains valid also for the complex stressed state, It is real/actual, let us determine the densities of additional work and potential of creep in the triaxial stressed state as follows:

$$\left.\begin{aligned} U &= \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \right.\\ &\qquad \left. - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] + \int_0^S g(S)\,dS,\\ \Phi &= \int_0^S v(S)\,dS. \end{aligned}\right\} \quad (21.5)$$

Here S - uniform function of the first degree from components of stress σ_1, σ_2, σ_3, introduced into §18.

Now expressions (18.1) for the components of the deformation rate will be recorded as follows:

$$\dot{e}_k = \frac{\partial}{\partial \sigma_k}\left(\frac{dU}{dt} + \Phi\right). \qquad (21.6)$$

The proof of the Castigliano theorem is conducted accurately in the same way as for frame. Let us assume that on body act generalized forces Q_s, which correspond the generalized displacement/movements will be q_s. Stresses are the functions of external forces, if external forces obtain increases δQ_s, the stresses at each point of body obtain increases $\delta\sigma_k$.

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The system of external forces δQ_s and of internal forces $\delta\sigma_k$ is counterbalanced, the field of true airspeeds of deformation $\dot{e}_k$ and of the velocities of the points of application of force $\dot{q}_s$ can be accepted as the field of virtual displacements; therefore from the beginning of virtual displacements, it follows:

$$\sum \dot{q}_s \delta Q_s = \int_V \dot{e}_k \delta\sigma_k \, dV = \int_V \frac{\partial}{\partial\sigma_k}\left(\frac{dU}{dt} + \Phi\right)\delta\sigma_k \cdot dV.$$

Integral in right side is taken by the volume of body. Let us assume

$$\int_V U \, dV = \tilde{U}, \quad \int_V \Phi \, dV = \tilde{\Phi}, \qquad (21.7)$$

then the equation of equilibrium is rewritten as follows:

$$\sum \dot{q}_s \delta Q_s = \delta\left(\frac{d\tilde{U}}{dt} + \tilde{\Phi}\right).$$

Since the stresses are expressed as external forces, values $\tilde{U}$ and $\tilde{\Phi}$ can be considered as functions Q_s; repeating the reasonings given above, we again come to formula (21.4), where $\tilde{U}$ and $\tilde{\Phi}$ are determined now by relationship/ratios (21.7).

The Castigliano theorem is applied for the determination of displacement/movements, and also for determining the excess unknowns in statically indeterminable systems. In the latter case the necessary equations are obtained via equating zero corresponding to derivative:

$$\frac{\partial}{\partial X}\left(\frac{d\tilde{U}}{dt}+\tilde{\Phi}\right)=0.$$

In the larger part of the cases, the actual determination of the stressed state in the body, subjected to creep, presents basic difficulty. For approximate solution of the tasks of creep useful is proved to be the following method. They are assigned stress distribution which it satisfies the equations of statics and it can be considered from one or the other considerations plausible. After this the rates of displacement/movements are found by the direct/straight application/use of Castigliano theorem. Similar examples will be given further.

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On conclusion of this paragraph, let us extract the formulas of the

potential of creep for the power and exponential law of creep:

a) the power law

$$v = e_n \left(\frac{S}{\sigma_n}\right)^n, \quad \Phi = \int v\, dS = \frac{e_n \sigma_n}{n+1} \left(\frac{S}{\sigma_n}\right)^{n+1}. \quad (21.8)$$

b) the exponential law

$$v = e_e \exp\left(\frac{S}{\sigma_e}\right), \quad \Phi = \int v\, dS = e_e \sigma_e \left[\exp\left(\frac{S}{\sigma_e}\right) - 1\right]. \quad (21.9)$$

§22. Determination of the service life of the elements of construction/design in the complex stressed state.

Experimental investigations of decomposition as a result of creep give grounds to consider that the cracking goes predominantly in the planes, perpendicular to the direction of maximum normal stresses. The rate of growth in the cracks should consider depending on the value of equivalent stress $\sigma_э$, undertaken, for example, in the form (18.8).

If is accepted for creep the criterion of greatest shearing stress, then the generalization of equations of type (15.1) to the case of the complex stressed state (§18) will be following system of equations:

$$\left.\begin{aligned} \dot{e}_1 &= -\dot{e}_3 = v\left(\frac{\sigma_1}{1-\omega} - \sigma_3\right), \\ \dot{e}_2 &= 0, \\ \dot{\omega} &= f(\sigma_э). \end{aligned}\right\} \quad (22.1)$$

Here φ - value of damage.

Solution of system (22.1) together with the equations of equilibrium and consistency of strain gives the service life of body under conditions of creep. However, this procedure is connected usually with sufficiently cumbersome calculations, which is not always justified, if is born in mind the considerable scatter of the characteristics of material. Therefore in the specific problems to preferably use rougher, but also substantially simpler estimates.

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In particular, the lower estimation of bearing capacity gives strength calculation according to the state of steady-state creep. It lies in the fact that is calculated the stressed state of steady-state creep and is determined the point or the totality of the points in which value

$$\sigma_э = \frac{1}{2}(\sigma_1 + \sigma_o)$$

reaches maximum, is located the value of value $\sigma_{э\,max}$ and is determined the time of the decomposition of the specimen/sample, elongated by constant stress $\sigma_{э\,max}$. This time is considered the time of structural failure.

The upper estimation of bearing capacity gives calculation

according to limiting condition. Limiting condition we call here such statically permissible stressed state of the evenly hot body, in which value $\sigma_э$ over critical section/cut is constant. In the case of variable over section/cut temperature, value $\sigma_э$ in limiting condition is such, that the time of decomposition in all points of critical section/cut is equal. The selection of critical section/cut in the practically important cases does not usually represent difficulties.

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Chapter III.

ELONGATION-COMPRESSION

§23. Determination of deformation at alternating loads and temperatures.

Let us assume that on the element of construction/design acts tensile (or compressive) stress σ, which is the function of time, temperature also is changed in time. The exemplary/approximate curve/graphs of function $\sigma(t)$ and $T(t)$ are depicted in Fig. 23. The definition of deformation as functions of time is reduced to solving of two separate tasks:

1. Determination of elasto-plastic deformation.

2. Determination of creep strain.

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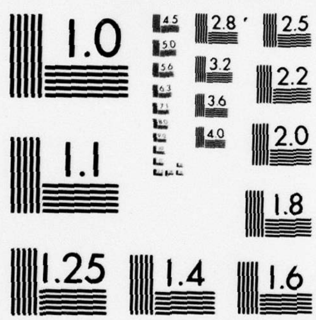

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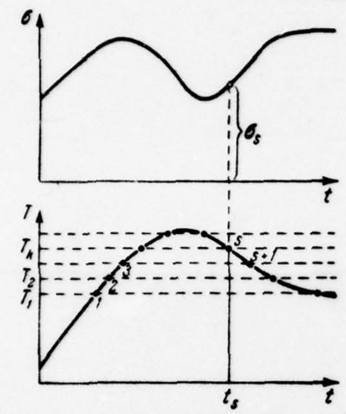

Fig. 23. Example of the programs of change σ and T in time.

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Determination of instantaneous elasto-plastic deformation. For this material is assumed to be given one the series of the curves of the instantaneous deformation $e_0(\sigma)$ for temperatures T_1, T_2, T_3, ... These curves are schematically depicted in Fig. 24, in chapter VI, are given actual curves for different materials. Let us note on axle/axis T of the graph/diagram of the dependence of temperature T on time (Fig. 23) those values T_k, for which there are curves of instantaneous deformation, let us conduct through the appropriate points horizontal lines, let us number the points of intersection these of straight lines with curved T(t) in ascending order t, as shown in drawing. At the moment of time t_s the temperature was T_k,

the corresponding stress, measured on the upper graph of Fig. 23, was σ_s. Now through instantaneous curved, appropriate temperature T_k, we find deformation e_{0s} for stress σ_s and we construct the appropriate point on the graph/diagram of dependence e_0 on t (Fig. 25). Repeating this construction for all points, we will obtain dashed curve. Deformation would follow this curved, if material was nonlinearly elastic.

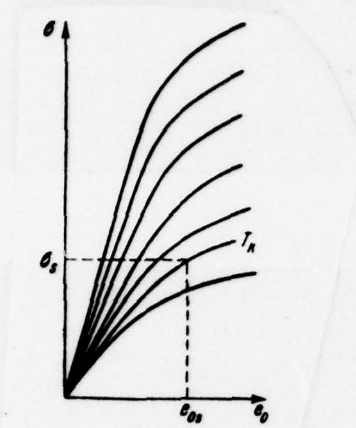

Fig. 24.

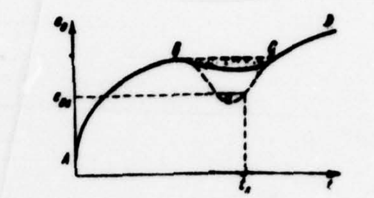

Fig. 25.

Fig. 24. To the determination of instantaneous elastc-plastic deformation. Series of curves σ-e_0 with different T.

Fig. 25. Dependence $e_0(t)$ during change σ and T in time according to Fig. 23.

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In actuality the diagram of instantaneous deformation describes only the process of active loading; therefore real sense make only sections AB and CD, on section BC, plastic deformation retains constant value, during the decrease of stress, is restored the elastic deformation and the distance of the corresponding point curved $e_0(t)$ on this section from the straight line BC is equal to $(\sigma_B - \sigma)/E$, where σ - stress at the moment of time in question, E - the

modulus of elasticity, depending on the temperature at this moment.

If, for example, at point D is produced unloading, residual deformation will be equal to the ordinate of point D in Fig. 25, minus of the elastic deformation, equal to $\sigma_D/E(T_D)$.

Determination of creep strain. Creep strain is located by numerical integration for the formula

$$p=\int_0^t v(\sigma,T)\,dt. \qquad (23.1)$$

Here σ and T are considered as functions of time.

Example of 23.1. Rod from the alloy EI-202 is subjected to loading according to the program, depicted in Fig. 26. The permissible permanent elongation of rod is 5c/c.

To determine the torque/moment of time t', when the plastic deformation of rod is proved to be equal permissible. If t'<500 s, then to correct the section/cut of rod then, so that t' would be equal to 500 s.

The curve of dependence $e_0(t)$ instantaneous plastic deformation on time is constructed according to of these Fig. 125 on the diagram of Fig. 27.

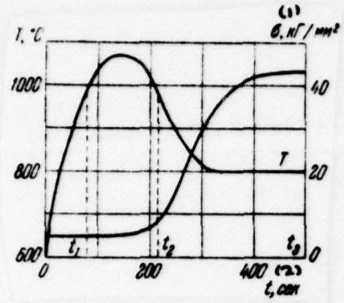

Fig. 26. Tc example of 23.1. Eredetermined programs T-t and σ-t.

Key: (1). kg/mm². (2). s.

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During the determinaticn of creep strains the true dependence n(T) it is tc convenient replace with certain approximation n'(T). Depicted in Fig. 28 the curve n(T), fcr example, can be replaced by the graph of the piecewise constant functicn. But in this case dependence $\sigma_n(T)$ should replace with dependence $\sigma_n'(T)$, which provides the best approximation cf the velccity cf creep in assigned/prescribed range of stress $\sigma_1 \leqslant \sigma \leqslant \sigma_2$. A precise dependence on stress, accordingly (14.3), exists

$$\lg \dot{p} = \lg e_n + n(T) \lg \frac{\sigma}{\sigma_n(T)}.$$

This dependence is replaced by that approximated:

$$\lg \dot{p} \approx \lg e_n + n'(T) \lg \frac{\sigma}{\sigma_n'(T)}.$$

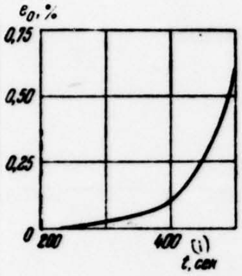

Fig. 27.

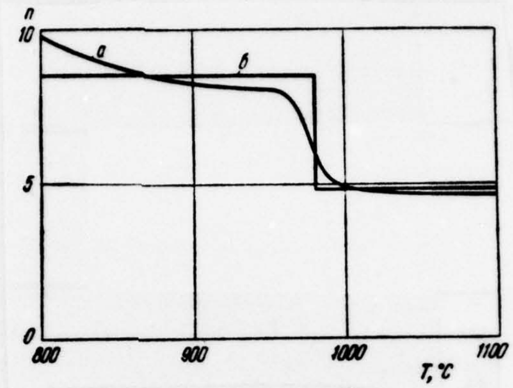

Fig. 28.

Fig. 27. To example of 23.1. Curve $\epsilon_0(t)$.

Key: (1). s.

Fig. 28. To example of 23.1. Approximation of dependence $n(T)$ by stepped diagram.

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The condition of the minimum of quadratic deviation in interval $\lg\sigma_1 \leqslant \lg\sigma \leqslant \lg\sigma_2$ leads to the following formula:

$$\lg\sigma'_n = \frac{n}{n'}\lg\sigma_n + \frac{\lg\sigma_1 + \lg\sigma_2}{2}\left(1 - \frac{n}{n'}\right). \qquad (*)$$

After accepting

$n'_1 = 8{,}50\ (T \leqslant 980°\,C)$ and $n'_2 = 4{,}80\ (T \geqslant 980°\,C)$

and after determining by formula (*) $\lg \sigma'_n$, it is reveal/detected that in our case $\lg \sigma'_n$ for all T it coincides with an accuracy to 1o/o with $\lg \sigma_n$. Therefore subsequently let us use $\lg \sigma_n$.

In table gives calculation curved p(t) for the assigned/prescribed loading. This curve is constructed in Fig. 29. Primes here carried out curve taking into account instantaneous plastic deformations (Fig. 27). On curve in Fig. 29 it is determined t* = 172 s.

(1) t, сек	T, °C	(2) σ, кГ/мм²	(2) σ_n, кГ/мм²	$\frac{\sigma}{\sigma_n}$	(3) $\dot{p}$, сек$^{-1}$	$\Delta p \cdot 10^2$	$p \cdot 10^2$
60	925	5,0	16,4	0,305	$4{,}17 \cdot 10^{-9}$	0,02	0
80	985	5,0	7,33	0,682	$1{,}60 \cdot 10^{-6}$	0,21	0,02
100	1031	5,0	4,37	1,143	$1{,}90 \cdot 10^{-4}$	0,74	0,23
120	1057	5,0	3,51	1,425	$5{,}50 \cdot 10^{-4}$	1,36	0,97
140	1070	5,0	3,24	1,545	$8{,}80 \cdot 10^{-4}$	1,64	2,33
160	1065	5,2	3,35	1,552	$8{,}27 \cdot 10^{-4}$	1,74	3,97
180	1052	5,8	3,65	1,588	$9{,}13 \cdot 10^{-4}$	1,47	5,71
200	1020	6,8	4,73	1,435	$5{,}60 \cdot 10^{-4}$	0,62	7,18
220	978	8,7	9,33	0,933	$5{,}57 \cdot 10^{-5}$	0,06	7,80
240	915	12,5	18,6	0,672	$0{,}34 \cdot 10^{-5}$	0,01	7,86
260	875	18,0	26,5	0,680	$0{,}37 \cdot 10^{-5}$	0,01	7,87
280	840	24,4	33,5	0,728	$0{,}67 \cdot 10^{-5}$	0,02	7,88
300	815	29,8	38,4	0,776	$1{,}16 \cdot 10^{-5}$	0,03	7,90
320	800	33,3	41,7	0,798	$1{,}47 \cdot 10^{-5}$	0,05	7,93
340	800	36,5	41,7	0,876	$3{,}22 \cdot 10^{-5}$	0,09	7,98
360	800	39,0	41,7	0,935	$5{,}55 \cdot 10^{-5}$	0,14	8,07
380	800	41,0	41,7	0,982	$8{,}55 \cdot 10^{-5}$	0,19	8,21
400	800	42,0	41,7	1,005	$1{,}04 \cdot 10^{-4}$	0,23	8,40
420	800	42,7	41,7	1,024	$1{,}22 \cdot 10^{-4}$	0,26	8,63
440	800	43,1	41,7	1,033	$1{,}34 \cdot 10^{-4}$	0,27	8,89
460	800	43,4	41,7	1,040	$1{,}40 \cdot 10^{-4}$	0,28	9,16
480	800	43,5	41,7	1,044	$1{,}44 \cdot 10^{-4}$	0,29	9,44
500	800	43,5	41,7	1,044	$1{,}44 \cdot 10^{-4}$		9,73

Key: (1). s. (2). kg/mm². (3). s^{-1}.

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Let us determine new value $\lambda = F_0/F$ (F_0 - cross-sectional area of this rod, F - cross-sectional area of the rod whose residual deformation it will be 5o/o). Let us note that the portion/fraction of instantaneous plastic deformation in our example proved to be small (Fig. 29) and it can be disregarded. Let us further break entire period of loading into three section: $0 \leqslant t \leqslant t_1 = 78$ s, where $T \leqslant 980$°C and creep can be disregarded; $t_1 < t \leqslant t_2 = 216$ s, where

$980° < T < 1100°$ and $n = n_2 = 4,80$ and $t_2 < t \leq t_3 = 500$ s, where $T \leq 980°$ and $n = n_1 = 8,50$. After designating the permissible deformation through [e], we can record

$$[e] = e_n \int_{t_1}^{t_2} \left(\frac{\lambda\sigma}{\sigma_n}\right)^{n_2} dt + e_n \int_{t_2}^{t_3} \left(\frac{\lambda\sigma}{\sigma_n}\right)^{n_1} dt = \lambda^{n_2} e_{12} + \lambda^{n_1} e_{23}.$$

Values e_{12} and e_{23} are determined from Fig. 29:

$$e_{12} = 7,75 \cdot 10^{-2},$$
$$e_{23} = 1,98 \cdot 10^{-2}.$$

Thus, we come to the equation

$$\lambda^{8,50} + 2,60\,\lambda^{4,80} = 1,675.$$

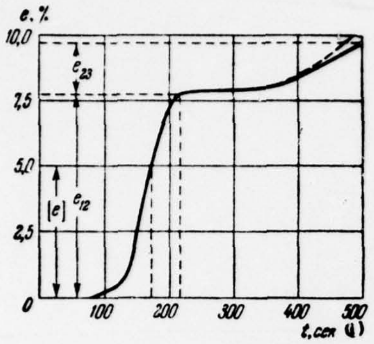

Fig. 29. To example of 23.1. Obtained diagram $\epsilon(t)$.
Key: (1). S.
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It is solved graphically in Fig. 30, where is obtained $\lambda_* = 0{,}873$. Consequently, the section/cut of rod must be increased by $(1/\lambda_* - 1)\cdot 100 = 14\%$.

Example 23.2. To determine the axial deformation of rod made of the stainless steel 1Kh18N10T which is compressed by constant stress σ=15 kg/mm² during 60 s at temperature of 800°C. Rod is compressed so that the lateral flexure is absent. Taking into account the observation, made into §20 relative to creep during compression, we will use characteristics for the elongation which are given into §52. Since creep of the stainless steel 1Kh18N10T at 800° - with strengthening, its characteristics are represented in the form of isochronal curves (Fig. 108). Let us find in this family of curves that, which corresponds to parameter t=60 s, and let us note on it the point, which corresponds σ=15 to kg/mm², deformation will be 2.75o/o.

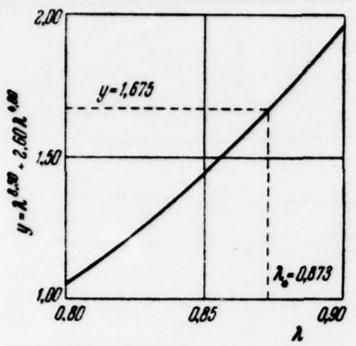

Fig. 30. To example of 23.1. Graphical solution of equations (*).

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§24. Determination of time to failure.

Fundamental fracture characteristic is introduced in §15 value e_*, which is conditional deformation at the moment of decomposition. At constant temperatures the criterion of decomposition is (15.4) $t_* = \frac{e_*}{n+1} \frac{1}{v(\sigma)}$. Thus, if value e_* is known, then for determining the torque/moment of decomposition is not required conductings of special calculations; after finding p(t), we determine this torque/moment from achievement condition by value p of certain critical value.

During rapid creep under conditions of the moderate temperatures, the decomposition bears connec for creep character and

is accompanied only by insignificant plastic deformation. In this case, during the determination of the torque/moment of decomposition, does not have the sense to consider the change in the cross-sectional area, connected with plastic deformation. At the sufficiently high temperatures (lower limit of this temperature range is stipulated in given below reference data due to separate alloys) it is possible to disregard embrittlement and to consider the process of decomposition during rapid creep purely viscous.

To inexpediently consider a change in the cross section as a result of plastic deformation, if value e_* does not exceed approximately 0.10. If e_* is more approximately 0.40, then it is possible to disregard embrittlement. At intermediate values one should consider both of these factors in timing of decomposition.

Since value e_* depends on temperature, then formulate the condition, similar (15.4) and valid at variable temperatures, which could easily be utilized in practical calculations, it is cannot. If one assumes that a change in the temperature does not have specific effect on the process of crack formation, i.e., to extend the principle of the linear addition of defectiveness to the case of variable temperatures, then decomposition condition for the power law of creep will be recorded in the form

$$\int_0^{t_*} \frac{n(T)+1}{e_*(T)} v(\sigma, T)\, dt = 1, \qquad (24.1)$$

where t_* - torque/moment of decomposition.

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However, in this case, the calculation of the service life of article it is better to perform directly on the curves of stress-rupture strength using the following method: the graph/diagrams of the dependence σ and T on time t are replaced by stepped curve/graphs, as shown in Fig. 31. In this case, one should select the ordinates of the step/stages of graph T(t) in such a way that they would coincide with the values of the temperatures for which we avail the curves of stress-rupture strength. This are determined the boundary/interfaces of time intervals. Let us designate through t_k the length of step/stage along the axis t, through T_k - the corresponding temperature, through σ_k - stress. On curved stress-rupture strength for temperature T_k we find the time of decomposition, which corresponds to stress σ_k, after designating this time $t_*^{(k)}$.

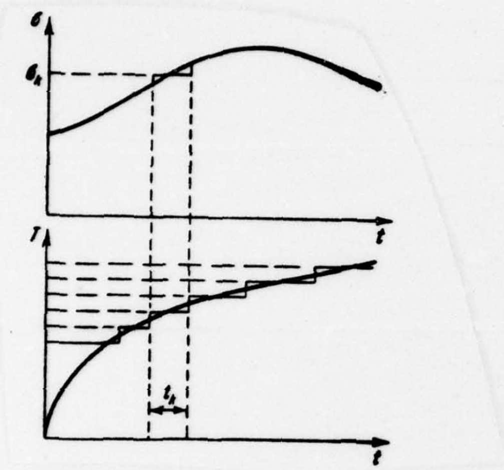

Fig. 31. Approximation cf graphs $\sigma(t)$ and $T(t)$ by stepped curve/graphs.

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We now compute values

$$\frac{t_1}{t_*^{(1)}},\quad \frac{t_1}{t_*^{(1)}}+\frac{t_2}{t_*^{(2)}},\quad \frac{t_1}{t_*^{(1)}}+\frac{t_2}{t_*^{(2)}}+\frac{t_3}{t_*^{(3)}},\ldots$$

and it is applied point in the coordinates

$$\sum_{i=1}^{k}\frac{t_i}{t_*^{(i)}} \quad \text{and} \quad t=\sum_{i=1}^{k} t_i.$$

Let us connect the points cf steady curved, the point of intersection this curved with the horizcrtal, which has the crdinate, equal to unity, will determine the time of decomposition t_* (Fig. 32).

Example 24.1. Thin-walled tube from the aluminum alloy D16T is loaded by the internal pressure q. Duct in running order is heated evenly to temperature of 325°C. Necessary service life of construction/design $t_* = 1000$ s. It is required to determine the permissible pressure, if tube bore D_0=20 mm, the thickness of wall δ_0=1 mm.

It is solved task taking into account embrittlement and thinning of duct in the process of creep.

The condition of volume constancy:

$$\pi D_0(1 + e)\delta = \pi D_0\delta_0.$$

e - deformation of the material of duct, δ_0 - initial thickness, δ - current thickness. The current stress in duct will be therefore

$$\sigma = \frac{q}{2}\frac{D_0}{\delta_0}(1+e)^2 = \sigma_0(1+e)^2.$$

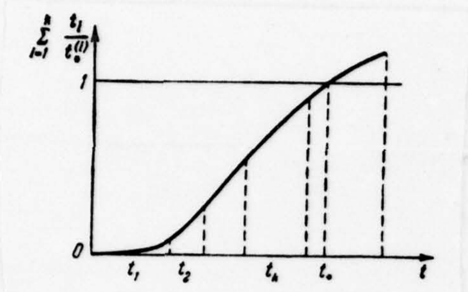

Fig. 32. To the determination of time to failure with variables σ and T.

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Deformation rate taking into account the embrittlement:

$$\frac{de}{dt} = \varepsilon_n \left(\frac{\sigma_0}{\sigma_n}\right)^n (1+e)^{2n} \left(1 - \frac{e}{e_*}\right)^{-n},$$

whence

$$t_* = \frac{1}{\varepsilon_n} \left(\frac{\sigma_n}{\sigma_0}\right)^n \psi(e_*, n),$$

where

$$\psi(e_*, n) = \int_0^{e_*} \frac{\left(1 - \frac{e}{e_*}\right)^n}{(1+e)^{2n}} de.$$

Values of the necessary constants for D16AT at 325°:

$e_* = 0{,}250, \quad n = 11{,}8, \quad \sigma_n = 8{,}35$ kg/mm².

Computation of integral gives $\psi = 0{,}01685$. The allowable stress in the

wall of duct will be

$$[\sigma_0] = \sigma_n \left(\frac{\psi}{e_n t_*}\right)^{1/n} = 7{,}19 \quad \text{kg/mm}^2 \qquad (*)$$

and the corresponding to it pressure $[q] = 2\frac{\delta_0}{D_0}[\sigma_0] = 71.9$ atm(tech).

In similar tasks, however, at the high values of n during the determination of bearing capacity it is possible to disregard a thinning of wall and an increase of the diameter of duct in the process of creep. It is real/actual, in that case we will obtain instead of (*), by utilizing (15.4):

$$[\sigma_0]' = \sigma_n \left(\frac{e_*}{n+1}\right)^{1/n} \left(\frac{1}{e_n t_*}\right)^{1/n}$$

and

$$\frac{[\sigma_0]'}{[\sigma_0]} = \left(\frac{e_*}{n+1}\frac{1}{\psi}\right)^{1/n} = 1{,}01.$$

Example 24.2. In the duct of example of 24.1 assigned/prescribed operating pressure q=110 atm(tech). It is required to determine the permissible temperature, if the necessary service life remains previous, $t_* = 1000$ s.

Taking into account the observation, made on conclusion of the preceding/previous example, let us determine the pressure, which corresponds to the assigned/prescribed service life, by disregarding an increase in the diameter of the duct:

$$q = 2\frac{\delta_0}{D_0}\sigma_n \left(\frac{e_*}{n+1}\right)^{1/n} \left(\frac{1}{e_n t_*}\right)^{1/n}.$$

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The results of calculation are given in table.

T, °C	(1) σ_n, кГ/мм²	n	e_*	(2) q, ат
275	15,30	17,1	0,240	136
300	11,35	14,3	0,235	99,5
325	8,35	11,8	0,250	71,9

Key: (1). kg/mm². (2). atm(tech)

The graph/diagram of dependence q(T) is given to Fig. 33. Here is determined temperature T_*, for which the permissible pressure is 110 atm(tech), it is equal to 293°C.

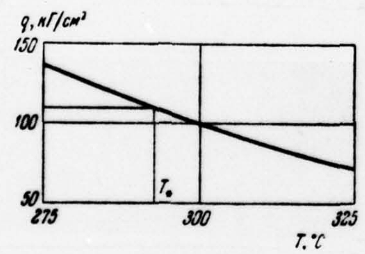

Fig. 33.

Fig. 33. To example of 24.2. Graphic determination of the permissible temperature.

Key: (1). kgf/cm².

§25. Simplest statically indeterminable tasks. Instantaneous deformation.

The formulation statically indeterminable tasks in an unavoidable manner is connected with the schematization of real construction/design and neglect of a number of factors, with difficulty which yield to account. Therefore, as a rule, during calculations for creep, there is no sense to examine the complex statically indeterminable systems whose full/total/complete investigation is connected with great computational difficulties.

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Here we will give full/total/complete solving only of simplest tasks, keeping in mind first of all the explanation of the degree of accuracy of some approximate procedures which it is necessary to recommend for practical use. The presentation of method is conveniently conducted immediately based on specific example.

Example 25.1. Figures 34 depicts statically indeterminable system, which consists of two rods from which is suspend/hung the rigid bar, to slewable around point by O. Material of rod 1 - D16AT, operating temperature T_1=250°C; the material of rod 2 - OT-4, temperature T_2=775°C. Datum temperature of both of rods, collected without assembly stresses, T_0=20°C, the cross-sectional area $F_1=F_2=F=50$ mm²; force P=1200 kgf.

Let us extract first in the table the necessary characteristics of the materials of rods.

(1) Номер стержня	(2) Материал	T, °C	(3) E, кГ/мм²	α
1	Д16АТ	250	$5{,}30 \cdot 10^3$	$24{,}8 \cdot 10^{-6}$
2	ОТ-4	775	$3{,}00 \cdot 10^3$	$8{,}5 \cdot 10^{-6}$

Key: (1). Number of rod. (2). Material. (3). kg/mm².

Figures 35 depicts instantaneous curved deformations for the material of rod 1 at temperature T_1 and the material of rod 2 at temperature T_2.

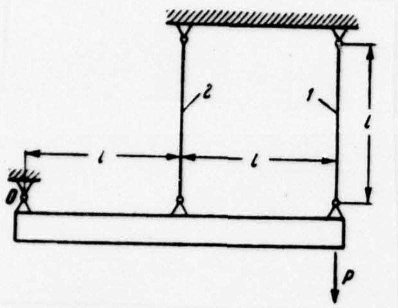

Fig. 34. To example of 25.1. Assigned/prescribed statically indeterminable system.

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We will consider that the compression stress-strain diagram coincides with stress-strain diagram.

The equation of statics can be recorded in the form

$$2\sigma_1 + \sigma_2 = 3\sigma, \qquad (25.1)$$

if is introduced the designation

$$\sigma = \frac{2P}{3F} = 16 \text{ kg/mm}^2.$$

The equation of the consistency of deformations will be following:

$$e_1 = 2e_2. \qquad (25.2)$$

First we will examine a question concerning the determination of the state of instantaneous deformation. In the tasks of plasticity,

is not indifferent the crder of the load application and not it is unimportant also, that cccurs more earlily, the load application or heating. Generally speaking, external lcad σ and temperature T must be assign/prescribed by the graphs, similar tc that given to Fig. 23, but if loading occurs during finite time, plastic strain is accompanied by creep, and it is necessary tc perform total calculation according tc the diagram which will be described in following paragraph.

Let us here examine scme simplest cases.

a) the determinaticn cf thermal stresses.

Let us determine first the thermal deformations

$$\theta_1 = \alpha_1 \Delta T_1 = 24{,}8 \cdot 10^{-6} \cdot 230 = 0{,}570 \cdot 10^{-2},$$
$$\theta_2 = \alpha_2 \Delta T_2 = 8{,}5 \cdot 10^{-6} \cdot 755 = 0{,}642 \cdot 10^{-2}.$$

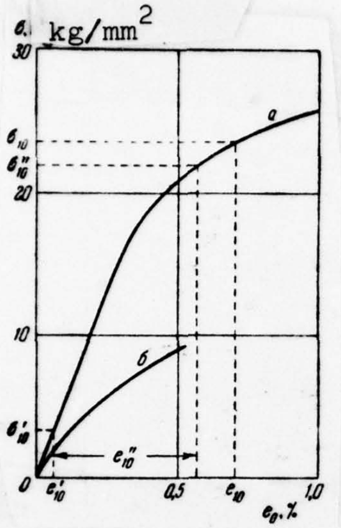

Fig. 35. To example of 25.1. Curves of instantaneous deformation for the materials of rods 1 and 2.

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Since $\theta_2 > (1/2)\theta_1$, then in rod 2 appear compressive stresses, in rod 1 - stretching. Let us designate through σ_2 the absolute value of compressive stress in the second rod, then equation (25.1) will take the following form:

$$2\sigma_1 - \sigma_2 = 0, \tag{25.3}$$

and equation (25.2)

$$\theta_1 + e_{10}(\sigma_1) = 2\theta_2 - 2e_{20}(\sigma_2)$$

or

$$\left.\begin{aligned} 2e_{20}(\sigma_2) + e_{10}(\sigma_1) &= 2\theta_2 - \theta_1, \\ e_{10}(\sigma_1) &= 2\theta - 2e_{20}(\sigma_2), \\ \theta = \theta_2 - \frac{1}{2}\theta_1 &= 0{,}642 \cdot 10^{-2} - \frac{1}{2}\, 0{,}570 \cdot 10^{-2} = \\ &= 0{,}357 \cdot 10^{-2}. \end{aligned}\right\} \tag{25.4}$$

Let us find first the voltages under the assumption of the elasticity of material. Then equation (25.4) will take the following form:

$$\frac{\sigma_1}{E_1} = 2\left(\theta - \frac{\sigma_2}{E_2}\right). \tag{25.5}$$

Solving this equation together with (25.3), we find

$$\sigma_1 = \frac{2\theta}{\frac{1}{E_1} + \frac{4}{E_2}} = \frac{2 \cdot 0{,}357 \cdot 10^{-2}}{\frac{1}{5{,}30 \cdot 10^3} + \frac{4}{3{,}00 \cdot 10^3}} = 4{,}69\ \kappa\Gamma/\text{мм}^2,$$

$$\sigma_2 = 2\sigma_1 = 2 \cdot 4{,}69 = 9{,}38\ \kappa\Gamma/\text{мм}^2.$$

Key: (1). kg/mm².

Since, in this case, our assumption about the elasticity of material proved to be inaccurate: σ_2=9.38 kg/mm² exceeds the value of limit of proportionality of alloy OT-4 at 775°C - that system of equations (25.4) and (25.3) should solve graphically, counting function e_{20} of given one graphically (Fig. 35).

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We are assigned by certain value σ_2 (see the table).

(1) σ_2 кГ/мм²	$e_{20} \cdot 10^2$	$e_{10} \cdot 10^2$	(1) σ_1 кГ/мм²
6,55	0,310	0,094	4,98
6,70	0,320	0,074	3,92
6,80	0,330	0,054	2,86

Key: (1). kg/mm².

we find on graph $e_{20}(\sigma_2)$ the appropriate value e_{20}, on formula (25.4) we compute $e_{10}=2(\theta-e_{20})$, set aside the obtained value e_{10} along the axis of abscissas on graph $e_{10}(\sigma_1)$, find value σ_{10}. It is

applied now the corresponding point in coordinates $\sigma_2-\sigma_1$ (Fig. 36). Repeating construction for different values σ_2, we will obtain depicted in Fig. 36 curve. We carry out now the ray/beam $\sigma_2=2\sigma_1$, which corresponds to equation (25.3), the point of intersection of this ray/beam from that found curved will determine values $\sigma_1=3.38$ kg/mm² and $\sigma_2=6.76$ kg/mm² after heating.

It is necessary to remember that σ_1 - tensile stress, and σ_2 - compressive. Let us note, that $e_{20}(\sigma_2)<0$; this condition determines upper boundary for σ_2 (Fig. 35, the curve b).

b). Determination of voltages from load application.

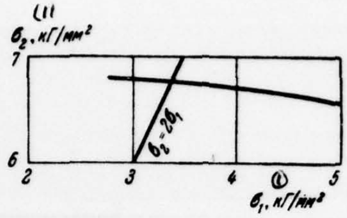

Fig. 36. To example 25.1. Graphic determination σ_1 and σ_2.

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The obtained thermal stresses are initial voltages for solving the next problem of the action of external force. Let us designate from σ'_{10} and σ'_{20}. From external force both of rods are dilate/extended, in this case, initial voltage σ'_{10} is positive, and σ'_{20} - it is negative. Therefore, subsequently we can use the previous diagram of dependence $e_{10}(\sigma_1)$ (Fig. 35a), and diagram $e_{20}(\sigma_2)$ one should supplement by the section, which corresponds to compression (Fig. 37).

Let us apply to these diagrams of point, which correspond σ'_{10} and σ'_{20}, now already with correct signs. If σ'_{20} does not exceed established/installed elastic limit, then with elongation dependence $e_{20}(\sigma_2)$ will follow the same diagram. If point σ'_{20} is located on the plastic section of the diagram (as this was obtained in our example), then it is necessary to construct diagram $e_{20}-\sigma_2$ for the

subsequent elongation taking into account the elastic unloading and the Bauschinger effect. This diagram is shown on Fig. 37 by dashed line. Further calculation is manufactured in perfect analogy that, as this was made during solving of temperature problem.

The equation of consistency (25.2) now connects the strains, calculated off the initial state of strain. Being assigned by certain arbitrary value σ''_{20}, we find value e''_{20} on Fig. 37, set aside $e''_{10}=2e''_{20}$ to curved a Fig. 35 and find appropriate σ''_{10}. Repeating this procedure, let us fill table and construct the graph/diagram of the dependence σ''_{10} on σ''_{20} (Fig. 38).

(1) σ''_{20} кГ/мм²	$e''_{20}\cdot 10^2$	$e''_{10}\cdot 10^2$	(1) σ''_{10} кГ/мм²
0,90	0,325	0,625	23,7
0,50	0,305	0,610	23,2
0	0,280	0,560	22,7

Key: (1). kg/mm².

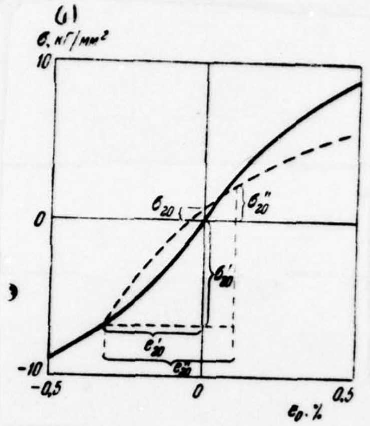

Fig. 37. To the determination of the voltage σ_{20} in example 25.1.

Key: (1). kg/mm².

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Intersection of this graph from straight line, determined by equation $2\sigma_1+\sigma_2=3\cdot16$, gives solving of the problem: $\sigma''_{10}=23,56$ kg/mm², $\sigma''_{20}=$ $=0,88$ kg/mm². Knowing voltages, we find through the diagrams of Fig. 35 (curve a) and 37 strains $e_{10}=0,72\cdot10^{-2}$, $e_{20}=0$. To them one should add thermal strains, and now it is possible to calculate the displacements of the which interest us points of system. So, displacement at the point of application of force will be

$$\delta_{10}=(e_{10}+\theta_1)\,l.$$

Angle of rotation of the rigid bar

$$\varphi_0=\frac{\delta_{10}}{2l}=\frac{1}{2}(e_{10}+\theta_1)=\frac{1}{2}(0,72\cdot10^{-2}+0,57\cdot10^{-2})=$$
$$=0,645\cdot10^{-2}.$$

Solving of the problem of the instantaneous deformations of system strongly is simplified, if it is possible to assume instantaneous behavior elastic. Then are applied the common impedance methods of materials, only it is necessary to consider the dependence of the modulus of elasticity on temperature.

Example 25.2. Two copper (M1) beaker/sleeves (Fig. 39) are connected by the rigid steel flanges (method of fastening flanges to beaker/sleeves on drawing is not shown).

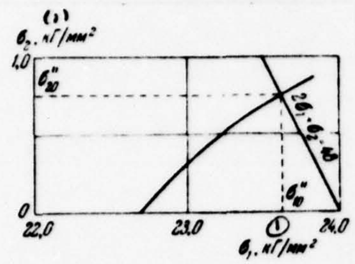

Fig. 38. To example 25.1. Graphic determination of voltages σ_{10} and σ_{20}.

Key: (1). kg/mm².

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Between beaker/sleeves - thermal insulation material bearing capacity of which can be disregarded. Construction/design is assembled without voltages at room temperature. To determine the voltages, which appear during the rapid heating of internal beaker/sleeve to T_1=600°C, and structural distortion, if the temperature of external beaker/sleeve remains room (T_2=20°). Cross-sectional area of the internal beaker/sleeve F_1=1500 of mm², external F_2=750 mm².

The condition of the consistency of strains taking into account the equation of equilibrium will be

$$\theta_1 - e_{10}(\sigma_1) = e_{20}(2\sigma_1).$$

Internal beaker/sleeve is compressed; therefore will cost to the left a difference in the absolute values of strains. Since $\theta_1=\alpha\cdot\Delta T=18\cdot10^{-6}\times580=1,04\cdot10^{-2}$, then

$$e_{10}(\sigma_1)+e_{20}(2\sigma_1)=1{,}04\cdot10^{-2}. \qquad (*)$$

Being assigned by the series of values σ_1, we determine on the curves of the instantaneous deformation of copper of value e_{10} and e_{20} (see the Table) and, after constructing the curve $e_{10}+e_{20}$ (Fig. 40), we find the solution of equation (*): $\sigma_{10}=1{,}825$ kg/mm².

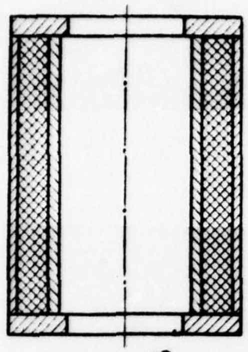

Fig. 39.

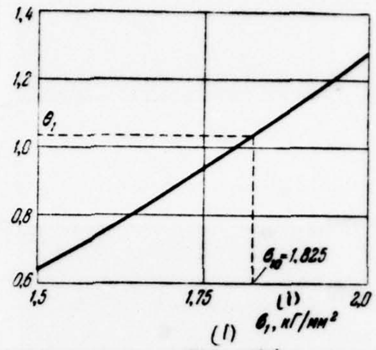

Fig. 40.

Fig. 39. Tc example 25.2. The schematic cf the node in which appear the thermal stresses.

Fig. 40. Tc example 25.2. Graphic determination of voltages σ_{10} and σ_{20}. Along the axis cf ordinates, it is plctted by $10^2(e_{10}+e_{20})$.

Key: (1). kg/mm².

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Consequently, $\sigma_{20}=3.65$ kg/mm² and the cverall elcngation per unit length of construction/design $e_0=e_{20}=0,64 \cdot 10^{-2}$.

(1) σ_1 кГ/мм²	$e_{10}\cdot 10^2$	(1) $2\sigma_1$ кГ/мм²	$e_{20}\cdot 10^2$	$(e_{10}+e_{20})\,10^2$
2,00	0,50	4,00	0,78	1,28
1,90	0,44	3,80	0,70	1,14
1,75	0,36	3,50	0,58	0,94
1,50	0,24	3,00	0,40	0,64

Key: (1). kg/mm².

Example of 25.3. The construction/design, examined in the preceding/previous example, after is heatable it is loaded by compressive force Q=7500 kg. To determine the voltages in both beaker/sleeves and the obtained structural distortion.

The curves of the instantaneous deformation of beaker/sleeves are given to Fig. 41.

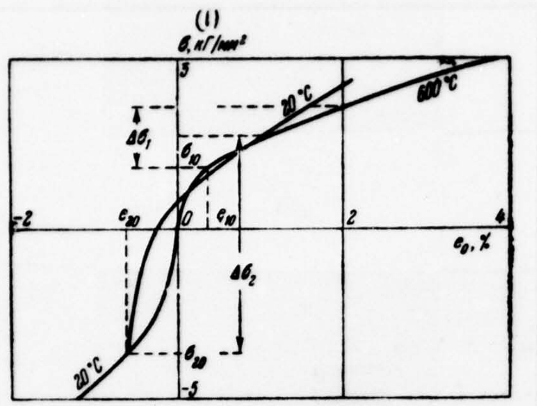

Fig. 41. To example 25.3. Curves of the instantaneous deformation of the beaker/sleeves, depicted in Fig. 39.

Key: (1). kg/mm².

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For an external beaker/sleeve the curve is constructed taking into account the Bauschinger effect. After designating $\sigma_0 = Q/F_1 = 5.00$ kg/mm², let us record the equation of equilibrium and the equation of the consistency of strains in the form

$$\Delta\sigma_1 + \frac{1}{2}\Delta\sigma_2 = \sigma_0,$$
$$\Delta e_{10}(\Delta\sigma_1) = \Delta e_{20}(\Delta\sigma_2).$$

This system is solved graphically (Fig. 42) with the use of data of following table.

$\Delta\sigma_1$ (1) $\kappa\Gamma/\text{мм}^2$	$\Delta e_{1\kappa} = \Delta e_{2\kappa} \cdot 10^2$	$\Delta\sigma_2$ (1) $\kappa\Gamma/\text{мм}^2$
1,00	0,82	4,90
1,25	1,08	5,45
1,50	1,37	5,90
1,75	1,64	6,40
2,00	1,94	6,85

Key: (1). kg/mm².

Results of solution following:

$$\Delta\sigma_1' = 1{,}77 \ \kappa\Gamma/\text{мм}^2,$$
$$\Delta\sigma_2' = 6{,}42 \ \kappa\Gamma/\text{мм}^2,$$
$$\sigma_1 = 3{,}60 \ \kappa\Gamma/\text{мм}^2,$$
$$\sigma_2 = 2{,}77 \kappa\Gamma/\text{мм}^2.$$

Key: (1). kg/mm².

The relative shortening of construction/design comprises $e=e_2=0{,}34$o/o.

§26. Statically indeterminable systems. State of steady-state creep.

From formula (13.2) it follows that creep strains increase in the course of time unlimitedly with any stress level, whereas instantaneous deformation is changed only because of a change in the stress and does not exceed certain specific in each case value.

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This means that with sufficiently long times t (conditionally large) instantaneous deformation can be disregarded and counted

$$e \approx p.$$

This limiting state is called the state of steady-state creep. If total strains are not too great, then the equations of statics can be record/written as before for the undeformed state of construction/design, in the examined (§25, Fig. 34) example of equation (25.1) it is retained:

$$2\sigma_1 + \sigma_2 = 3\sigma. \quad (26.1)$$

The equation of consistency (25.2) also is retained, but to us to conveniently use this equation, it differentiated it preliminarily on the time:

$$\dot{e}_1 = 2\dot{e}_2. \qquad (26.2)$$

Let us now record the law of creep, by disregarding instantly the strain:

$$\dot{e}_1 = v_1(\sigma_1), \quad \dot{e}_2 = v_2(\sigma_2). \qquad (26.3)$$

System of equations (26.1), (26.2), (26.3) is sufficient for determining of unknown voltages σ_1 and σ_2 and the unknown rates $\dot{e}_1$ and $\dot{e}_2$; so, under power law we obtain from (26.2) and (26.3)

The jcint solution of equations (26.1) and (26.4) determines voltages. For this, it is pcssitle tc reccmmend graphic method, in accuracy/precision the repeating that methcd which was used for the determination of instantanecus defcrmation.

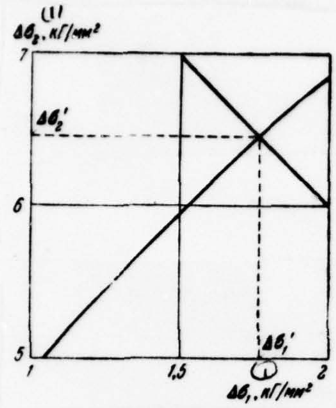

Fig. 42. To example 25.3. Graphic determination of voltages $\Delta\sigma'_1$ and $\Delta\sigma'_2$.

Key: (1). kg/mm².

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Is constructed the graph/diagram of dependence σ_2 on σ_1 according to equation (26.4), on this graph is conducted the straight line, that corresponds to equation (26.1), point of intersection determines the stressed state of steady-state creep. The corresponding calculation is given in table, on Fig. 43 is found the point of intersection **curved (26.4) and straight line (26.1):** $\sigma_1'' = 22{,}48$ **kg/mm² and** $\sigma_2'' = 3{,}05$ **kg/mm²** $(n_1 = 15{,}3,\ \sigma_{n1} = 17{,}7$ **kg/mm²,** $n_2 = 3{,}0,\ \sigma_{n2} = 1{,}15$ **kg/mm²,** $\varepsilon_{n1} = \varepsilon_{n2} = \varepsilon_n = 10^{-4}$ **s⁻¹).**

(1) σ_2 кГ/мм²	$\left(\frac{\sigma_2}{\sigma_{n2}}\right)^{n_2}$	$\left(\frac{\sigma_1}{\sigma_{n1}}\right)^{n_1}$	(1) σ_1 кГ/мм²
2,0	5,25	10,5	20,6
2,5	10,3	20,6	21,6
3,0	17,8	35,6	22,4
3,5	28,0	56,0	23,0

Key: (1). kg/mm².

The free-running speed of the rotation of the rigid bar:

$$\dot{\varphi}'' = \dot{e}_2'' = \varepsilon_n\left(\frac{\sigma_2}{\sigma_{n2}}\right)^{n_2} = 10^{-4}\left(\frac{3,65}{1,15}\right)^3 = 0,32\cdot 10^{-2} \text{ rad/s.}$$

If index n for two rods is identical, problem is solved in an explicit form. For simplicity let us consider that other constants of creep are also identical, although this does not have fundamental value. Thus, let us assume

$$n_1 = n_2, \quad \varepsilon_{n1} = \varepsilon_{n2}, \quad \sigma_{n1} = \sigma_{n2}.$$

Then from (26.4)

$$\sigma_1 = \sigma_2 \cdot 2^{1/n},$$

and from (26.1):

$$\sigma_1 = \frac{3\cdot 2^{1/n}}{2^{\frac{n+1}{n}}+1}\sigma, \quad \sigma_2 = \frac{3}{2^{\frac{n+1}{n}}+1}\sigma. \qquad (26.5)$$

With n=1 hence follows:

$$\sigma_1 = \frac{6}{5}\sigma, \quad \sigma_2 = \frac{3}{5}\sigma.$$

This is the case of linear ductility/toughness/viscosity. The

same the stress distribution will be in elastic rod.

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With n=∞ we obtain

$$\sigma_1 = \sigma_2 = \sigma.$$

The voltages in rods are identical, if we consider the material of rods perfectly plastic and to solve the problem of the limiting condition of system. It is turned out that always, by solving the problem of steady-state creep under the power law of the dependence of velocity on stress and after making passage to the limit with n ∞, we will obtain the same distribution of stresses, which corresponds to the limiting condition of system from perfectly plastic cell/elements. Difference lies in the fact that the limiting condition is possible only at the specific value of external load, respectively the stresses in rods are equal to yield point, i.e., to certain constant, which depends on the material of rod. The voltages in rods during creep to very high index n are identical, but their value is determined by external load. Let us note, that with increase n of stress level σ_1 and σ_2 they approach value $\sigma_1=\sigma_2=\sigma$, which is obtained with n=∞. Let us note also, that the great voltage σ_1 monotonically diminishs with increase n. The ratio of actual value σ_1 to value σ, which is obtained in the extreme case with n=∞, is function n moreover, if n is changed from n=1 to n=∞, then

$$\frac{6}{5} \geqslant \frac{\sigma_1}{\sigma} \geqslant 1.$$

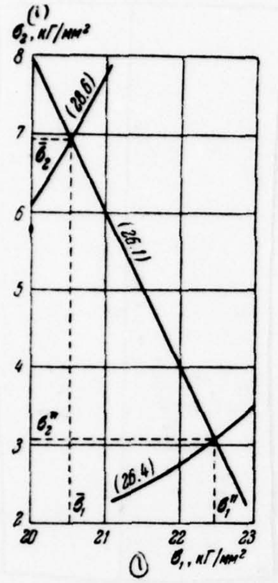

Fig. 43. Graphic determination of the voltages σ_1 and σ_2 in the rods of the system, presented in Fig. 34.

Key: (1). kg/mm².

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In this case, this sense can be call/named the stress concentration factor. In all examined, until now, problems it was turned out that the concentration factor diminishs with increase n.

Example 26.1. Plate from titanium alloy OT-4 is supported by two angle irons made of the stainless steel EI-696. Panel is

dilate/extended by force of P. The permissible elongation of panel for time t'=400 s composes Δl= 10 mm. To determine the permissible force of P depending on temperature (in the range of temperatures of 750-1000°C). Size/dimensions of panel l=500 mm, b=200 mm, δ=2 mm, F=150 mm² (Fig. 44).

Let us find the permissible force, by disregarding the unsteady stage of creep. Equation of the equilibrium

$$2\sigma_1 F_1 + \sigma_2 b\delta = P$$

and equation of the consistency of the strains

$$\dot{e}_1 = \dot{e}_2 = \dot{e}$$

give taking into account communication/connection between the deformation rates and the stresses

$$P = 2\sigma_{n1}\left(\frac{\dot{e}}{\varepsilon_n}\right)^{1/n_1} F_1 + \sigma_{n2}\left(\frac{\dot{e}}{\varepsilon_n}\right)^{1/n_2} b\delta.$$

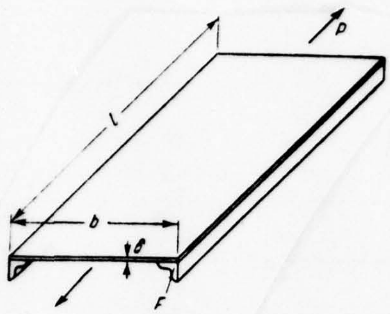

Fig. 44. Schematic of panel to example of 26.1.

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If into this expression is substituted for $\dot{\varepsilon}$ the allowed value of the rate of deformation $[\dot{e}] = \Delta l / lt' = 10/500 \cdot 400 = 5 \cdot 10^{-5}$ s^{-1}, then force P will obtain the maximum permissible value. Further calculation is represented in table (index $n_2=3$ in all temperature range; therefore, $\left(\frac{\dot{e}}{\varepsilon_n}\right)^{1/n_2} = 0{,}794$).

T °C	n_1	(1) σ_{n1} кГ/мм	(1) $\left(\frac{\dot{e}}{\varepsilon_n}\right)^{1/n_1}$	(1a) σ_{n2} кГ/мм²	(2) P_1 кГ	(2) P_2 кГ	(2) P кГ
750	12,35	42,8	0,945	2,18	12 140	694	12 834
800	9,20	26,0	0,927	0,78	7 230	248	7 478
850	8,10	14,45	0,918	0,51	3 980	162	4 142
900	7,28	8,15	0,910	0,46	2 220	146	2 366
950	5,85	5,82	0,877	0,39	1 530	124	1 654
1000	4,95	4,27	0,870	0,29	1 110	92	1 202

Key: (1). kg /mm. (1a) kg/mm². (2). kg .

Example 26.2. Frame, depicted in Fig. 45, is loaded by force of P, applied to absolutely rigid beam. Material and the temperature of all rods are identical. To determine, at which point must be applied force P, so that the steady motion of beam will be forward/progressive.

Equations of the equilibrium:

$$N_1 + N_2 + N_3 = P, \qquad (*)$$
$$(2N_2 + 3N_1)a = Px. \qquad (**)$$

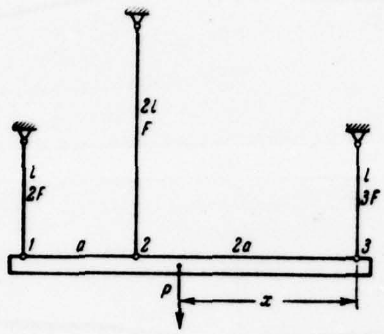

Fig. 45. To example 26.2. Schematic of assigned/prescribed frame.

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The condition of forward motion consists of equality the rates of the displacement of points 1, 2, 3:

$$v_1 = v_2 = v_3,$$

that taking into account the dependence of creep rates on stresses it will be recorded in the form

$$\left(\frac{N_1}{2F\sigma_n}\right)^n l = 2\left(\frac{N_2}{F\sigma_n}\right)^n l = \left(\frac{N_3}{3F\sigma_n}\right)^n l,$$

or

$$\frac{1}{2} N_1 = 2^{1/n} N_2 = \frac{1}{3} N_3. \qquad (***)$$

Joint solution of equations (*), (**), (***) gives

$$x = \frac{2 + 3\cdot 2^{1+\frac{1}{n}}}{1 + 2^{1+\frac{1}{n}} + 3\cdot 2^{\frac{1}{n}}}\, a.$$

With

$$n \to \infty \qquad x \to \frac{4}{3} a.$$

§27. The approximation methcd of the study cf steady-state creep.

During rapid creep in the larger part cf the cases, index n is prcved to be large and the stress distributicn in the elements statically indeterminable system differs little from the stress distribution in the limiting ccnditicn of perfectly plastic system. For approximate computaticns it is possible tc recommend to accept the voltages in cell/elements identical ones. Sc one should enter when the constants of creep of all cell/elements are identical. Let us look how will be matter when these ccnstants are different. Returning to an old example (Fig. 34), let us record (26.4) in the following form:

$$\frac{\sigma_1}{\sigma_{n1}} = 2^{1/n_1}\left(\frac{\sigma_2}{\sigma_{n2}}\right)^m, \quad m = \frac{n_2}{n_1}. \qquad (27.1)$$

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Let us examine now the following versicns:

1. $n_2 >> n_1$, m - large number. Set/assuming $m=\infty$, let us find that:

a) $\sigma_1=0$, if $\sigma_2 < \sigma_{n2}$,

b) σ_1 is not defined, if $\sigma_2 = \sigma_{n2}$,

c) $\sigma_1=\infty$, if $\sigma_2>\sigma_{n2}$.

Real ones are only cases a) and b). With light loads is realize/accomplished case a), from (26.1) it follows:

$$\sigma_2=3\sigma, \quad \sigma_1=0 \quad (3\sigma<\sigma_{n2}).$$

This means that the creep strength of the first rod is so small that it can be disregarded, and it is necessary to consider that entire/all load is absorbed by the second rod. At the high values of load, when $3\sigma>\sigma_{n2}$, we must accept

$$\sigma_2=\sigma_{n2}, \quad \text{consequently,} \quad \sigma_1=\frac{3\sigma-\sigma_{n2}}{2}.$$

2. $n_1>>n_2$, m is completely small. Let us assume m=0; then from (27.1) it follows that

$$\sigma_1=\sigma_{n1}, \quad \text{if} \quad \sigma_2\neq0,$$

$$\sigma_1<\sigma_{n1}, \quad \text{if} \quad \sigma_2=0.$$

Position is proved to be in accuracy/precision the same as with the second version, only rods 1 and 2 are changed by fields. We obtain

$$\sigma_1 = \frac{3}{2}\sigma, \qquad \sigma_2 = 0\left(\frac{3}{2}\sigma < \sigma_{n1}\right),$$
$$\sigma_1 = \sigma_{n1}, \qquad \sigma_2 = 3\sigma - 2\sigma_{n1}\left(\frac{3}{2}\sigma > \sigma_{n1}\right).$$

As is evident, constant σ_n plays role, in certain sense similar to the role of yield point in the problems of the theory of limit equilibrium.

3. $n_2=n_1>>1$, $m=1$. From (27.1) it follows:

$$\frac{\sigma_1}{\sigma_{n1}} = \frac{\sigma_2}{\sigma_{n2}},$$

or

$$\sigma_1 = \lambda\sigma_{n1}, \qquad \sigma_2 = \lambda\sigma_{n2}.$$

From the equation of equilibrium, it follows:

$$\lambda(2\sigma_{n1} + \sigma_{n2}) = 3\sigma.$$

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Therefore

$$\sigma_1=\frac{3\sigma\sigma_{n1}}{2\sigma_{n1}+\sigma_{n2}},\qquad \sigma_2=\frac{3\sigma\sigma_{n2}}{2\sigma_{n1}+\sigma_{n2}}.$$

The dependence of the speed of creep strain on voltage under power law for the different values of index n is depicted in Fig. 46. With increase n, the curves unlimitedly approach the broken line OAB, which corresponds to value $n=\infty$. With increase n, becomes narrow that range of voltages, in which is detected creep, with $n\to\infty$ the speed is equal to zero, if $\sigma<\sigma_n$, and speed is infinitely great, if $\sigma>\sigma_n$, thus, σ_n it is possible to identify with creep limit. Actually we have always a matter with the finite values of n; therefore for transition to $n=\infty$, it is necessary to regard at the procedure of approximation, useful during the solution of some tasks.

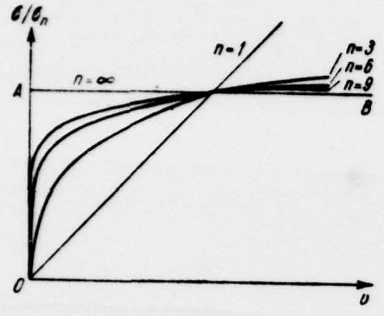

Fig. 46. Dependence of the speed of creep strains on voltage with different n.

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§28. Determination of the time of the decomposition statically indeterminable system.

Full/total/complete system of equations for solving the problem indicated includes: the equations of equilibrium, condition of the consistency of strains and communication/connection between creep rate and voltage taking into account embrittlement, accepted, for example, in the form

$$\dot{e} = \dot{e}_n\left(\frac{\sigma}{\sigma_n}\right)^n\left(1 - \frac{e}{e_*}\right)^{-n}, \qquad (28.1)$$

where e_* - strain with rupture.

Simpler will be estimations according to the state of steady-state creep and according tc limiting ccndition. In the case cf evenly heated frame whcse all cell/elements are made from one material, limiting condition ccincides with the stressed state in the case of ideal plasticity (with an accuracy tc ccnstant factor).

Let us explain aforesaid based on examples.

Example 28.1. Tc determine the time of the decomposition statically indeterminable ccnstruction/design, depicted in Fig. 34, if the material of rods and their temperature are identical.

After using the obtained above solution fcr steady-state creep (26.5), let us compare the voltage in the first, more stressed rod with the equation of curved stress-rupture strength (15.4). We will cbtain

$$t'_* = \frac{1}{n+1}\,\frac{e_*}{e_n}\left(\frac{\sigma_n}{\sigma}\right)^n \frac{1}{2}\left(\frac{2^{1+\frac{1}{n}}+1}{3}\right)^n. \qquad (28.2)$$

If voltages are are determined according tc limiting condition, then $\sigma_1=\sigma_2=\sigma$ (see §26) and

$$t''_* = \frac{1}{n+1}\,\frac{e_*}{e_n}\left(\frac{\sigma_n}{\sigma}\right)^n. \qquad (28.3)$$

We will obtain now the solution, by taking into account the redistribution of voltages, connected with the third section of curve of creep. For each of the rods, can be recorded equation (28.1). From the equation of **the consistency of strains (25.2):**

$$\sigma_1\left(1-\frac{e_2}{e_*}\right)=2^{1/n}\sigma_2\left(1-\frac{e_1}{e_*}\right).$$

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Let us eliminate from this relationship/ratio σ_2 with the aid of the equation of statics (25.1) and e_2 with the aid of (25.2), we will obtain

$$\sigma_1=\frac{3\cdot 2^{1/n}\left(1-\frac{e_1}{e_*}\right)}{\left(1-\frac{e_1}{2e_*}\right)+2^{1+\frac{1}{n}}\left(1-\frac{e_1}{e_*}\right)}\sigma.$$

After substituting this expression into equation (28.1) for the first rod, let us have

$$\frac{de_1}{dt}=\varepsilon_n\left(\frac{\sigma}{\sigma_n}\right)^n\frac{2^{n+1}3^n}{\left[2\left(1+2^{1+1/n}\right)-\left(1+2^{2+1/n}\right)\frac{e_1}{e_*}\right]^n}. \quad (28.4)$$

We integrate equation (28.4) when at the initial moment was $e_1=0$, at moment t_* of the decomposition of the first **rod:** $e_1=e_*$. **We will obtain the following formula for the critical time:**

$$t_*=\frac{e_*}{\varepsilon_n}\left(\frac{\sigma_n}{\sigma}\right)^n\frac{\left(1+2^{1+1/n}\right)^{n+1}-2^{-(n+1)}}{3^n(n+1)\left(1+2^{2+1/n}\right)}. \quad (28.5)$$

In table gives the relative values of critical time with

different n. As unit accepted critical time, calculated on formula (28.5), which one should consider most precise. Thus, in the second of table gives relations to the time, calculated on formula (28.2), to the time, calculated on formula (28.5).

n	t'_*/t_*	σ'_*/σ_*	t''_*/t_*	σ''_*/σ_*
1	0,9091	0,9091	1,0909	1,0909
2	0,8714	0,9335	1,0706	1,0348
3	0,8583	0,9505	1,0630	1,0205
5	0,8484	0,9675	1,0572	1,0110
8	0,8429	0,9787	1,0564	1,0070

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As usual, estimating strength according to stress distribution, found for steady-state creep, we obtained the decreased value of time to failure; error goes into safety factor. With increase n, this error grow/rises; however, from this it does not follow in any way that the calculation according to the steady stage with increase n becomes less precise. Designer usually deals with the permissible load. But if is calculated the voltages, which correspond to the assigned/prescribed service life, according to formulas (28.2) and (28.5) is are taken the relations of these voltages, then we will obtain the numbers, given in the third column of table. As is evident, the error in the determination of dangerous voltage is decreased with an increase in the index and becomes less than 5o/o, beginning approximately with n=5.

In the fourth and fifth columns are given also the data on the comparison of the forecasts of formulas (28.3) and (28.5). Calculation according tc limiting condition, upper estimation of service life (or bearing capacity with the assigned/prescribed service life), gives considerably greater accuracy/precision than calculation according tc the state of steady-state creep without taking into account of the third section of creep.

Example 28.2. Tc determine the service life of the construction/design of example of 25.1 (Fig. 34).

Let us find the upper estimation of service life according to limiting condition. Let us extract first the dependence of rupture stress from time. For titanium alloy OT-4 (rod 2) by embrittlement during rapid creep it is possible to disregard (see page 194); therefore its service life with elongation is determined by Hoff:

$$t_*^{(2)} = \frac{1}{e_n n_2}\left(\frac{\sigma_{n2}}{\sigma_2}\right)^{n_2}.$$

For the material of rod 1 (D16AT) the service life is determined taking into account the embrittlement:

$$t_*^{(1)} = \frac{e_*}{e_n(n_1+1)}\left(\frac{\sigma_{n1}}{\sigma_1}\right)^{n_1}, \qquad e_* = 0{,}240.$$

We solve now the equation of equilibrium (25.1)

$$2\sigma_1 + \sigma_2 = 3\sigma \qquad (*)$$

under condition $t_*^{(1)} = t_*^{(2)} = t_*''$. We have

$$\frac{1}{n_2}\left(\frac{\sigma_{n2}}{\sigma_2}\right)^{n_2} = \frac{e_*}{n_1+1}\left(\frac{\sigma_{n1}}{\sigma_1}\right)^{n_1}. \quad (**)$$

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The point of intersection of straight line (*) and curved, that corresponds to the equation

$$\sigma_2 = \sigma_{n2}\left(\frac{n_1+1}{e_* n_2}\right)^{1/n_2}\left(\frac{\sigma_1}{\sigma_{n1}}\right)^{n_1/n_2} = 3{,}25\left(\frac{\sigma_1}{17{,}7}\right)^{5{,}1}, \quad (28.6)$$

to equivalent condition (**), is obtained graphically in Fig. 43:

$$\bar{\sigma}_1 = 20{,}54\ \kappa\Gamma/\text{мм}^2 \ (1), \quad \bar{\sigma}_2 = 6{,}94\ \kappa\Gamma/\text{мм}^2 \ (1).$$

Key: (1). kg/mm².

Consequently, the upper estimation of service life will be

$$t_*'' = \frac{1}{\varepsilon_n n_2}\left(\frac{\sigma_{n2}}{\sigma_2}\right)^{n_2} = \frac{1}{10^{-4}\cdot 3}\left(\frac{1{,}15}{6{,}94}\right)^3 = 15{,}3 \text{ s}.$$

Lower estimation corresponds to the time of the decomposition of the first rod in the voltage, which occurs in the state of steady-state creep (in example 25.1 σ_1=22,48 kg/mm²):

$$t_*' = \frac{e_*}{\varepsilon_n(n_1+1)}\left(\frac{\sigma_{n1}}{\sigma_1}\right)^{n_1} = \frac{0{,}240}{10^{-4}(15{,}3+1)}\left(\frac{17{,}7}{22{,}48}\right)^{15{,}3} = 3{,}8 \text{ s}.$$

Example 28.3. To determine the permissible load q for

statically indeterminable frame, depicted in Fig. 47, if the necessary service life at temperature of 800°C is 1000 s. Construction/design is heated evenly. Part E is rigid. Material of rods - steel EI-696, the cross-sectional area of each rod F=50 of mm². Size/dimensions of the construction/design: h=100 mm, $\alpha=30°$, $\beta=45°$.

Let us find first the voltages in rods in the state of steady-state creep.

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The sum of moments with respect to point O will be

$$\sum M_O = N_2 h \cos\beta + N_1 h \cos\alpha - \frac{1}{2}\frac{qh^2}{\mathrm{tg}\,\alpha},$$

and consequently,

$$\sigma_1 \cos\alpha + \sigma_2 \cos\beta = \sigma_0. \qquad (*)$$

where

$$\sigma_0 = \frac{1}{2}\frac{qh}{F\,\mathrm{tg}\,\alpha} = \sqrt{3}q.$$

The equation of the consistency of strains we will obtain, after comparing two positions of rigid bar B (Fig. 48). Considering displacement/movements small ones, so that changes in the angles α and β can be disregarded, we can record for the rates of the elongation of the rods

$$v_1 = \frac{\mathrm{tg}\,\beta}{\mathrm{tg}\,\alpha} v_2,$$

so that, by utilizing **a power law cf creep, we will** obtain

$$\sigma_1 = \left(\frac{\mathrm{tg}\,\beta}{\mathrm{tg}\,\alpha}\right)^{1/n} \sigma_2. \qquad (**)$$

Thus, with final n we **have $\sigma_1 > \sigma_2$ and, disregarding** the redistribution cf voltages as a result of embrittlement, let us estimate the service life of construction/design according tc the time of the decomposition of the **first rod.** Frcm (*) and (**) we have

$$\sigma_1 = \frac{\sigma_0}{\cos\alpha + \cos\beta\left(\frac{\mathrm{tg}\,\alpha}{\mathrm{tg}\,\beta}\right)^{1/n}} = \frac{\sqrt{3}\,q}{\cos 30^\circ + \cos 45^\circ\left(\frac{\mathrm{tg}\,30^\circ}{\mathrm{tg}\,45^\circ}\right)^{1/n}}.$$

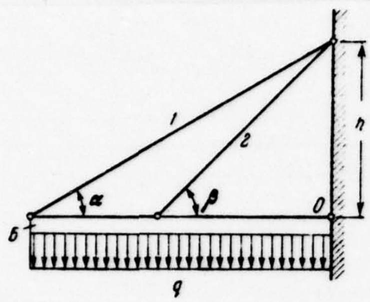

Fig. 47. Diagram of given one in an example 28.3 frames.

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In our case n=9,20 and $\sigma(1000)$=31,6 kg/mm². Therefore q=9,30 kg/mm².

Let us determine now ultimate load, by accepting the stress distribution in rods according to limiting condition. Then from equation (*) with $\sigma_1=\sigma_2=\sigma$ we obtain

$$\sigma = \frac{\sigma_0}{\cos\alpha + \cos\beta} = \frac{\sqrt{3}\,q}{\cos 30^\circ + \cos 45^\circ} = \sigma(1000) = 31{,}6 \text{ kg/mm}^2$$

and q=9.53 kg/mm².

Thus, the difference between these two solutions proved to be lesser than 3o/o. This it was to be expected for the case, when n=9.20.

Example 28.4. The construction/design, depicted in Fig. 49, consists of two duralumin (D16T) rods by the total area F=100 of mm² and one rod of titanium alloy VT- 14 with the same area F=100 mm². Temperature of duralumin rods of 300°, titanium 700°. Part A can be mixed only forward/progressively. To determine the time of structural failure, if P=3200 kgf.

Let us find first the state of steady-state creep. Equation of the equilibrium

$$\sigma_1+\sigma_2=\sigma_0, \qquad \sigma_0=\frac{P}{F}=32 \text{ kg/mm}^2.$$

Equation of the consistency of the strains

$$\left(\frac{\sigma_2}{\sigma_{n2}}\right)^{n_2}=\left(\frac{\sigma_1}{\sigma_{n1}}\right)^{n_1}$$

or

$$\frac{\sigma_2}{\sigma_{n2}}=\left(\frac{\sigma_1}{\sigma_{n1}}\right)^{n_1/n_2}.$$

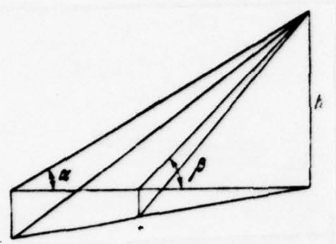

Fig. 48. To example 28.3. Diagram, which illustrates the elongation of frame.

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Necessary constants:

$$n_1 = 14{,}35, \qquad \sigma_{n1} = 11{,}35 \overset{(1)}{\kappa\Gamma/\text{мм}^2},$$
$$n_2 = 4{,}80. \qquad \sigma_{n2} = 17{,}4 \overset{(1)}{\kappa\Gamma/\text{мм}^2}.$$

Key: (1). kg/mm².

Consequently, $n_1/n_2=m=2.99$, and it is pcssible to accept $m=3$. Then system of equations will be

$$\sigma_2 = 0{,}0118\sigma_1^3,$$
$$\sigma_1 + \sigma_2 = 32,$$

and equation for σ_1 will be

$$\sigma_1^3 + 84{,}8\sigma_1 - 2710 = 0.$$

Solving this equation, we obtain

$$\sigma_1 = 11{,}9 \ ^{(1)}\kappa\Gamma/\text{мм}^2,$$
$$\sigma_2 = 20{,}1 \ ^{(1)}\kappa\Gamma/\text{мм}^2.$$

Key: (1). kg/mm².

If is disregarded the redistribution of voltages as a result of embrittlement, then the service life of construction/design should be estimated according to the time of the decomposition of rods from D16T, since for this alloy (with T=300°) e_* =0,235, and the embrittlement of titanium alloy VT-14 with T=700° can be disregarded.

Consequently, the service life of construction/design will be

$$t'_* = \frac{1}{1+n}\frac{e_*}{e_n}\left(\frac{\sigma_{n1}}{\sigma_1}\right)^{n_1} = \frac{1}{14{,}35+1}\,\frac{0{,}235}{10^{-4}}\left(\frac{11{,}35}{11{,}90}\right)^{14{,}35} = 79 \text{ s.}$$

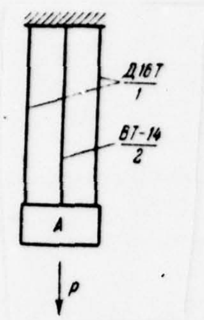

Fig. 49. Diagram of given one in an example of 28.4 statically indeterminable systems.

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§29. Total calculation statically indeterminable system to creep taking into account instantaneous deformation.

When us interests precise knowledge of the law of the redistribution of the voltages in the elements statically indeterminable system in dependence on time, and the extents of movements we want to determine with possible accuracy/precision, one should resort to the procedure of numerical calculation which is described below. Let us take for certainty the same simplest statically indeterminable system which was examined above (Fig. 34). Let us assume,that as a result of certain program of short-term deformation in rods are reached strain e'_1 and e'_2, to which

correspond the voltages σ'_1 and σ'_2. The prccedure of the determination of these stresses and strains was described into §25; if plastic deformation in the process of initial loading money-changer is sign, then it is necessary to consider the Bauschinger effect, by transferring correspcndingly the curve of deformation in parallel tc himself. Let us ccnstruct the appropriate diagrams as this done in Fig. 50, and let us ncte the points, which correspond to initial state. It is solved now the task of steady-state creep, i.e., let us determine the vcltages σ''_1 and σ''_2 cn the assumption that the instantaneous plastic deformation is absent. Let us note values σ''_1 and σ''_2 on the axle/axes of the ordinates of the diagrams of Fig. 50. In the process of creep, will proceed the redistribution of vcltages from σ'_1 to σ''_1 in the first rod and from σ'_2 to σ''_2 the secondly. As is evident, voltage σ_1 is decreased, in rod 1, occurs elastic unloading, the instantaneous deformation, which accompanies creep, follows Hooke's law, i.e., it goes over straight line a, indicated in Fig. 50, in the direction of arrow/pointer. In the second rod the vcltage increases; therefore creep is accompanied by additional plastic deformation in accordance with the curve of deformaticn.

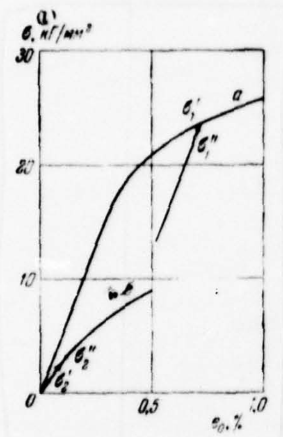

Fig. 50. To the determination cf the redistribution of the voltages in the process of creep in the rods of the system, presented in Fig. 34.

Key: (1). kg/mm².

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Calculation let us perform as follows. Let us break the range of change in the voltage σ_2 frcm $\sigma_2=\sigma'_2$ to $\sigma_2=\sigma''_2$ on k of parts, the appropriate stress levels let us designate $\sigma_{20}=\sigma'_2, \sigma_{21}, \ldots, \sigma_{2i}, \ldots, \sigma_{2k}=\sigma''_2$. These intervals must not be equal ones, it is convenient to take them smaller with an increase of σ_2. To each value σ_{2i} corresponds stress level in first rod $\sigma_{1i}=\frac{3\sigma-\sigma_{2i}}{2}$ in accordance with the equation of statics. Thus, it proved to be broken into sections also the range of

change in the voltage σ_1. As a result of the law of creep

$$\Delta e_{2i} = \Delta (e_{02})_i + e_{n2} \int_{t_i}^{t_{i+1}} \left(\frac{\sigma_2}{\sigma_{n2}}\right)^{n_2} dt. \qquad (29.1)$$

Here $\Delta(e_{02})_i$ - a change in the instantaneous deformation during a change in the voltage σ_2 from value σ_{2i} to value $(\sigma_2)_{i+1}$. If is noted the points, which correspond $\sigma_i, \sigma_{i+1}, \ldots$, to curved b Fig. 50, then values $\Delta(e_{02})_i$ are located by direct measurement on drawing. Let us now consider approximately that in range from σ_i to σ_{i+1} the voltage is changed in time according to linear law, so that

$$\sigma_2 = \sigma_{2i} + \frac{\Delta\sigma_{2i}}{\Delta t_i}(t - t_i).$$

Here t_i - the time when $\sigma_2 = \sigma_{2i}, \Delta t_i = t_{i+1} - t_i$. Under this hypothesis

$$\int_{t_i}^{t_{i+1}} \sigma^n dt = \frac{1}{n+1} \frac{\sigma_{i+1}^{n+1} - \sigma_i^{n+1}}{\Delta\sigma_i} \Delta t_i = \frac{1}{n+1} \frac{\Delta(\sigma_i^{n+1})}{\Delta\sigma_i} \Delta t.$$

As thus the (considering $e_{n1} = e_{n2} = e_n$),

$$\Delta e_{2i} = \Delta (e_{02})_i + \frac{1}{n_2+1} \frac{e_n}{\sigma_{n_2}^{n_2}} \frac{\Delta(\sigma_{2i}^{n_2+1})}{\Delta\sigma_{2i}} \Delta t_i. \qquad (29.2)$$

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It is analogous for the first rod

$$\Delta e_{1i} = \frac{\Delta\sigma_{1i}}{E_1} + \frac{1}{n_1+1} \frac{e_n}{\sigma_{n_1}^{n_1}} \frac{\Delta(\sigma_{1i}^{n_1+1})}{\Delta\sigma_{2i}} \Delta t_i. \qquad (29.3)$$

Let us introduce now (29.2) and (29.3) into the equation of the consistency

$$\Delta e_i = 2\,\Delta e_2$$

is solved the obtained equation **relatively** Δt_i. **We will** obtain

$$e_n\,\Delta t_i = \frac{2\Delta\,(e_{02})_i - \dfrac{\Delta\sigma_{1i}}{E_1}}{\dfrac{1}{n_1+1}\cdot\dfrac{1}{\sigma_{n1}^{n_1}}\,\dfrac{\Delta\left(\sigma_{1i}^{n+1}\right)}{\Delta\sigma_{1i}} - \dfrac{2}{n_2+1}\,\dfrac{1}{\sigma_{n2}^{n_2}}\,\dfrac{\Delta\left(\sigma_{2i}^{n_2+1}\right)}{\Delta\sigma_{2i}}}. \quad (29.4)$$

We lower sufficiently bulky calculation according to formula (29.4) and is given final result in the form of Fig. 51.

In the case $n_1=n_2=n$ **and** $\sigma_{n1}=\sigma_{n2}=\sigma_n$ **equation** (29.4) is simplified

$$e_n\,\Delta t_i = (n+1)\,\sigma_n^n\,\frac{2\Delta\,(e_{02})_i - \dfrac{\Delta\sigma_{1i}}{E}}{\dfrac{\Delta\left(\sigma_{1i}^{n+1}\right)}{\Delta\sigma_{1i}} - \dfrac{2\Delta\left(\sigma_{2i}^{n+1}\right)}{\Delta\sigma_{2i}}}. \quad (29.5)$$

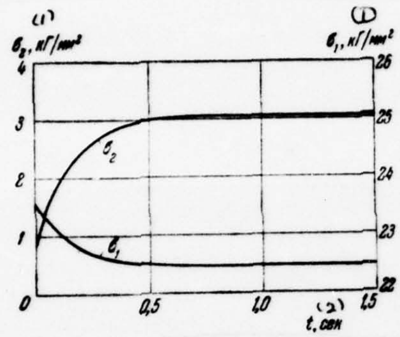

Fig. 51. Change in the time of the voltages in rods in the system, presented in Fig. 34.

Key: (1). kg/mm². (2). s.

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Approximately it is possible to accept

$$\Delta\left(\sigma_i^{n+1}\right) = \sigma_{i+1}^{n+1} - \sigma_i^{n+1} \approx (n+1)\,\sigma_i^n\,\Delta\sigma_i. \qquad (29.6)$$

Then formula (29.5) is replaced by following, simpler:

$$e_n\,\Delta t_i = \frac{2\Delta\left(e_{02}\right)_i - \dfrac{\Delta\sigma_{1i}}{E}}{\left(\dfrac{\sigma_{1i}}{\sigma_n}\right)^n - 2\left(\dfrac{\sigma_{2i}}{\sigma_n}\right)^n}. \qquad (29.7)$$

From this formula is directly perceived, which when $\sigma_{1i}=\sigma_1''$ and $\sigma_{2i}=\sigma_2''$ stops Δt=∞, thus, to the state of steady-state creep corresponds infinite time. During practical calculations the use of formula (29.5) does not introduce essential complications in

comparison with (29.7), but gives considerably high accuracy/precision and it makes it possible to select a relatively small number of step/pitches during calculation.

Example 29.1. The rods statically indeterminable system (Fig. 52) are made from D16AT and have a cross-sectional area F=100 [redacted] mm². Construction/design is assembled with T=20° without the tension of rods and is heated to T_1=350°C. Then to it is applied load P=1200 [redacted] kgf. To determine changes in the voltages in rods and the displacement/movement of node C in time, disregarding the temperature expansion of basis/base. l=200 mm, α=30°.

Condition of equilibrium at point C:

$$\sigma_1 + \sqrt{3}\sigma_2 = \sigma, \quad \sigma = \frac{P}{F} = \frac{1200}{100} = 12 \text{ kg/mm}^2. \qquad (*)$$

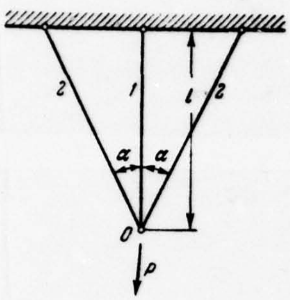

Fig. 52. Assign/prescribed in example 29.1 statically indeterminable frame.

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Condition of the consistency of deformations at point O:

$$e_1 = \frac{e_2}{\cos^2 \alpha} = \frac{4}{3} e_2,$$

i.e. after the heating

$$e_{10} + \theta_1 = \frac{4}{3}(e_{20} + \theta_2). \qquad (**)$$

For D16AT: $\alpha(20-400°C)=25,5 \cdot 10^{-6}$ deg^{-1}, E(350°C=3,10•10^3 kg/mm^2 and

$$\theta = \theta_1 = \theta_2 = 25,5 \cdot 10^{-6} \cdot 330 = 0,84 \cdot 10^{-2}.$$

Assuming that the thermal stresses in rods (σ_{1T} and σ_{2T}) do not exceed limit of proportionality, let us record equations (*) and (**) in the form

$$\sigma_{1T} + \sqrt{3}\sigma_{2T} = 0,$$
$$3\sigma_{1T} - 4\sigma_{2T} = \theta E,$$

whence

$$\sigma_{2T} = -\frac{\vartheta E}{4+3\sqrt{3}} = -\frac{0{,}84\cdot 10^{-2}\times 3{,}10\cdot 10^{3}}{4+3\sqrt{3}} = \\ = -2{,}83 \overset{(1)}{\kappa\Gamma/\text{мм}^2},$$

$$\sigma_{1T} = -\sqrt{3}\,\sigma_{2T} = -\sqrt{3}(-2{,}83) = 4{,}90 \overset{(1)}{\kappa\Gamma/\text{мм}^2}.$$

Key: (1). kg/mm². Assuming that and voltages, which arose directly following by load application; they are also elastic ones, we lead system of equations of equilibrium and consistency of deformations to form

$$\sigma_1' + \sqrt{3}\sigma_2' = 12,$$

$$(\sigma_1' - \sigma_{1T}) = \frac{4}{3}(\sigma_2' - \sigma_{2T}).$$

Solution of the system:

$$\sigma_1' = 10{,}76 \overset{(1)}{\kappa\Gamma/\text{мм}^2},$$

$$\sigma_2' = 0{,}72 \overset{(1)}{\kappa\Gamma/\text{мм}^2},$$

Key: (1). kg/mm².

and, therefore, assumption about the absence instantaneous plastic deformations proved to be correct.

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Let us note here that generally at the sufficiently high temperatures (what is 350° for an aluminum alloy) rapid creep with speeds to 10^{-2} s^{-1} it goes, as a rule, against the background only of elastic deformations. Instantaneous plastic deformations appear with

high voltages and at creep rates.

The displacement/movement of node C after heating and load application will be

$$\Delta x_0 = l(\theta + e_1) = 200\left(0{,}84 \cdot 10^{-2} + \frac{10{,}76}{3{,}10 \cdot 10^3}\right) = 2{,}38 \text{ mm.}$$

In the state of the steady-state creep:

$$\sigma_1'' + \sqrt{3}\,\sigma_2'' = 12, \qquad n = 8{,}65,$$
$$\left(\frac{\sigma_1''}{\sigma_n}\right)^n = \frac{4}{3}\left(\frac{\sigma_2''}{\sigma_n}\right)^n, \qquad \sigma_n = 5{,}70.$$

Solution of this system:

$$\sigma_1'' = 4{,}50 \overset{(1)}{\kappa\Gamma/\text{мм}^2},$$
$$\sigma_2'' = 4{,}34 \overset{(1)}{\kappa\Gamma/\text{мм}^2}.$$

Key: (1). kg/mm².

The free-running speed of the displacement/movement of node O:

$$\dot{x}_0 = le_n\left(\frac{\sigma_1''}{\sigma_n}\right) = 200 \cdot 10^{-4}\left(\frac{4{,}50}{5{,}70}\right)^{8{,}65} = 2{,}57 \cdot 10^{-3} \text{ mm/s.}$$

For transition from initial state to that being steady:

$$\Delta e_{1i} = \frac{\Delta\sigma_{1i}}{E} + \frac{1}{n+1}\frac{\varepsilon_n}{\sigma_n^n}\frac{\Delta\left(\sigma_{1i}^{n+1}\right)}{\Delta\sigma_{1i}}\Delta t_i,$$

$$\Delta e_{2i} = \frac{\Delta\sigma_{2i}}{E} + \frac{1}{n+1}\frac{\varepsilon_n}{\sigma_n^n}\frac{\Delta\left(\sigma_{2i}^{n+1}\right)}{\Delta\sigma_{2i}}\Delta t_i.$$

The equation of the consistency of deformaticns remains previous, and we obtain

$$\Delta t_i = \frac{(n+1)\,\sigma_n^n}{E\varepsilon_n}\,\frac{3\,\Delta\sigma_{1i} - 4\,\Delta\sigma_{2i}}{4\,\frac{\Delta\left(\sigma_{2i}^{n+1}\right)}{\Delta\sigma_{2i}} - 3\,\frac{\Delta\left(\sigma_{1i}^{n+1}\right)}{\Delta\sigma_{1i}}}.$$

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Replacing for the difference in the degrees of their approximations (29.6), we will obtain

$$\Delta t_i = \frac{1}{E\varepsilon_n} \frac{3\,\Delta\sigma_{1i} - 4\,\Delta\sigma_{2i}}{4\left(\frac{\sigma_{2i}}{\sigma_n}\right)^n - 3\left(\frac{\sigma_{1i}}{\sigma_n}\right)^n} = \frac{1}{E\varepsilon_n} \frac{1{,}77\,\Delta\sigma_{1i}}{1{,}33\left(\frac{\sigma_{2i}}{\sigma_n}\right)^n - \left(\frac{\sigma_{1i}}{\sigma_n}\right)^n}. \quad (*)$$

The corresponding increases in the displacement/movement of node 0 will be

$$\Delta x_i = l_1 \varepsilon_n \left(\frac{\sigma_{1i}}{\sigma_n}\right)^n \Delta t_i = 0{,}02 \left(\frac{\sigma_{1i}}{\sigma_n}\right)^n \Delta t_i \text{ s.} \quad (**)$$

The results of calculation according to formulas (*) and (**) are given to Fig. 53. As is evident, steady state is reached completely rapidly.

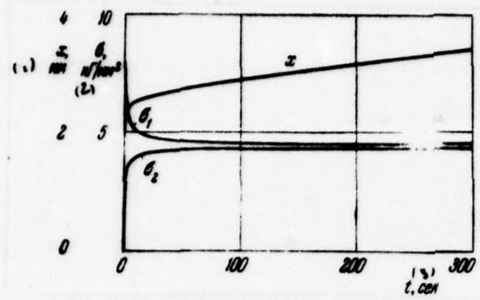

Fig. 53. To example 29.1. Change in the time of the voltages in the rods of system.

Key: (1). mm. (2). kg/mm². (3). s.

§30. Relaxation tasks.

By stress relaxation it is accepted to call the process of redistributing the initial voltages, created with the collection of construction/design or which appear as a result of heating. If external forces are absent, then solving of the task of steady-state creep gives for voltages in all cell/elements zero values, thus the process of relaxation lies in the fact that the voltage vanishes. The method of solving the relaxation tasks actually in no way differs from the method, presented into §29.

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Initial state for calculation for relaxation corresponds to zero external force, and if $\sigma=0$, then σ'_1 and σ'_2 compulsorily they must have different signs. They will have different signs, also, in the process of relaxation, each diminishing in absolute value, thus instantaneous deformation is elastic, and there is no need for knowing the diagram of instantaneous deformation, it is necessary only in order to find the initial state, which depends on the order of the assembly of construction/design and program of its heating. For application/appendices is of interest the special case of relaxation task when there is only one cell/element, which is found in the state of creep, connected with elastic cell/elements. The corresponding diagram is given to Fig. 54, the creeping cell/element is connected in series with the spring whose rigidity exist C. Let the length of rod be equal l, and the area of its section/cut F. For the assembly of construction/design, it is necessary to elongate spring to value u_0 and to connect with the end of the rod. Spring, striving to be reduced, will stretch rod, in it will arise the voltage σ_0, to which corresponds the elastic elongation $\sigma_0 l/E$; thus, the drawing of spring at the initial moment will be $u_0-\sigma_0 l/E$ and the corresponding force is equal to

$$P=\left(u_0-\frac{\sigma_0 l}{E}\right)C=\sigma_0 F. \qquad (30.1)$$

In the process of relaxation, the rod obtains deformation e(t), its

elongation at each moment of time $\Delta l = le(t)$, therefore, the elongation of spring exists $u_0 - le$, and force is equal to

$$P = (u_0 - le)C = \sigma F.$$

Eliminating hence u_0 with the aid of (30.1), we will obtain

$$\sigma = \sigma_0\left(1 + \frac{\beta}{E}\right) - \beta e. \qquad (30.2)$$

Fig. 54. Model construction/design, working under conditions stress relaxation.

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Here

$$\beta = \frac{Cl}{F}.$$

If β is small, then of (30.2) it follows $\sigma \approx \sigma_0$, we have creep with constant stress; if β is completely great, then $e = \sigma_0/E$=const. Last/latter condition, this condition of pure relaxation,, which occurs, when the ends of the rod are securely fastened. Under actual conditions pure relaxation never is encountered, although precisely this case they attempt to carry out with larger or smaller degree of accuracy during relaxation tests. Let us examine the task for relaxation under the arbitrary law of creep. Differentiating from time (30.2) let us find

$$\dot{e} = -\frac{\dot{\sigma}}{\beta}.$$

Let us introduce this expression into the fundamental equation of

creep (13.1), taking into account that σ diminishs and, therefore, $\varkappa=0$. We will obtain

$$-\frac{1}{E_n}\dot{\sigma}=v(\sigma), \qquad \frac{1}{E_n}=\frac{1}{\beta}+\frac{1}{E}.$$

Hence follows:

$$t=\left(\frac{1}{\beta}+\frac{1}{E}\right)\int\limits_{\sigma}^{\sigma_0}\frac{d\sigma}{v(\sigma)}. \qquad (30.3)$$

Let us examine the special cases:

a). Power law

$$\varepsilon_n t=\left(\frac{1}{\beta}+\frac{1}{E}\right)\sigma_n^n\frac{1}{n-1}\left[\frac{1}{\sigma^{n-1}}-\frac{1}{\sigma_0^{n-1}}\right]. \qquad (30.4)$$

As is evident, voltage is turned into zero with t=∞.

b). Exponential law

$$\varepsilon_e t=\left(\frac{1}{\beta}+\frac{1}{E}\right)\sigma_e\left\{\exp\left(-\frac{\sigma}{\sigma_e}\right)-\exp\left(-\frac{\sigma_0}{\sigma_e}\right)\right\}. \qquad (30.5)$$

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With the use of this formula, it is necessary to remember that the exponential law of creep is accurate only for not too small voltages and is completely incorrect when $\sigma/\sigma_e \ll 1$. Therefore it must not astonish the fact that on formula (30.5) it is obtained, seemingly to value σ=0 corresponds finite time of relaxation. In order to obtain qualitatively correct result, one should use the law of hyperbolic sine.

c). Law of hyperbolic sine

$$e_e t = 2\left(\frac{1}{\beta} + \frac{1}{E}\right)\sigma_e \ln \frac{\operatorname{th} \sigma_0/2\sigma_e}{\operatorname{th} \sigma/2\sigma_e}.$$

Example 30.1. Flanges A and B are drawn together by bolts (Fig. 55). Initial voltage in bolts σ_0=17,0 kg/mm². Material of flanges and bolts EI-696, temperature of 900°. To determine the time, for which the voltage in bolts will fall 4 times.

It is possible to accept flange distortions A and B only elastic ones, since the voltages in them at least in $\frac{\pi}{4}[(2d)^2 - d^2] \Big/ \frac{\pi}{4} d^2 = 3$ time are less than in bolts, if are considered that work only the ranges, noted on drawing by dot-and-dash line, and to disregard the bend of flanges. Therefore the creep rate of the material of flanges will be in $3^n = 3^{7,28}$ time it is less than the rate of deformation of bolts.

Consequently, it is possible to use equation (30.4), after assuming in it $\beta=\infty$ and $\sigma=\sigma_0/4$, then

$$t = \frac{\sigma_n^n}{e_n E} \frac{1}{n-1} \frac{4^{n-1}}{\sigma_0^{n-1}} = \frac{8,16^{7,28}}{10^{-4} \cdot 0,9 \cdot 10^4} \frac{1}{7,28-1} \frac{4^{7,28-1}}{17^{7,28-1}} = 89 \text{ s}.$$

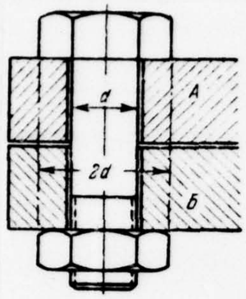

Fig. 55. Diagram of assigned/prescribed in an example 30.1 bolted joint.

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Example 30.2. Flanges with bolts - the same as in example of 30.1. Material of flanges and bolts EI-696. Temperature increases with speed θ=2 deg/s. To determine the dependence of the voltage in bolts from time, if initial voltage σ_0=10,0; 15.0; 20.0 kg/mm².

With the same assumptions that in example of 30.1, it is possible to record

$$\frac{\sigma}{E} + g(\sigma_{max}) + p = \text{const}.$$

Accepting, that the module/modulus of elasticity $E=0.9 \cdot 10^4$ kg/mm² does not depend on temperature, we will obtain

$$\frac{\dot{\sigma}}{E} + \varepsilon_n \left(\frac{\sigma}{\sigma_n}\right)^n = 0,$$

$$\frac{d\sigma}{dT} = -\frac{\varepsilon_n E}{\dot{\vartheta}} \left(\frac{\sigma}{\sigma_n}\right)^n = -0,45 \left(\frac{\sigma}{\sigma_n}\right)^n.$$

The results of integrating this equaticn by the method of Euler - Cauchy with the subsequent iterations are given to Fig. 56. From curve/graph it is evident that, beginning with T=890°, the voltages in bolts dc not depend on initial voltage.

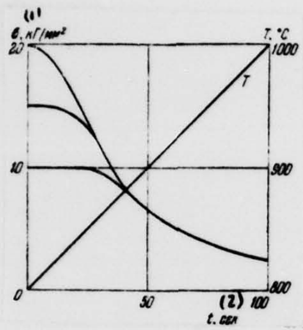

Fig. 56. To example 30.2. Change in the voltage in bolt in time.

Key: (1). kg/mm². (2). s.

§31. Solving of the tasks of steady-state creep under exponential law.

During calculations to steady-state creep the law of creep, i.e., the form of the dependence of creep rate on voltage, can be any. So, in the examined example (Fig. 34) the equation of consistency (26.2) reduces to the following equation, which connects the voltages:

$$v_1(\sigma_1) = 2v_2(\sigma_2). \qquad (31.1)$$

In the general case - this is the transcendental equation which always can be solved graphically or numerically. The application/use of an exponential law of creep sometimes has advantages which we will now see.

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Let us assume that the constants of the creep of the material of rods are different, so that

$$v_1(\sigma_1)=\varepsilon_e'\exp\left(\frac{\sigma_1}{\sigma_e'}\right),\qquad v_2(\sigma_2)=\varepsilon_e''\exp\left(\frac{\sigma_2}{\sigma_e''}\right).$$

Substituting in (31.1), let us lead the equation of consistency to the following form:

$$\exp\left(\frac{\sigma_1}{\sigma_e'}-\frac{\sigma_2}{\sigma_e''}\right)=\frac{2\varepsilon_e''}{\varepsilon_e'},$$

or

$$\frac{\sigma_1}{\sigma_e'}-\frac{\sigma_2}{\sigma_e''}=\gamma. \qquad (31.2)$$

Here

$$\gamma=\ln\frac{2\varepsilon_e''}{\varepsilon_e'}.$$

System (31.2) and (26.1) is the system of linear equations whose solution is located immediately. It is necessary only to bear in mind, that the obtained result makes sense when $\sigma_1>\sigma_e',\ \sigma_2>\sigma_e''$, otherwise the application/use of an exponential law justified, but also creep in this case is unessential, it it is possible into calculation not to accept. If voltages are sufficiently large, then the right side of equation (31.2) is small in comparison with separate ones add/composed by left, and we can accept instead of (31.2) the following approximate equation of consistency:

$$\frac{\sigma_1}{\sigma_e'}=\frac{\sigma_2}{\sigma_e''},$$

or

$$\sigma_1 = \lambda \sigma'_e, \qquad \sigma_2 = \lambda \sigma''_e.$$

Furthermore as under power law (§27), it is turned out that the constants σ'_e and σ''_e play the role of yield points and the task of steady-state creep stops the similar to task theory of the limit equilibrium of the ideally-plastic system.

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For examined previously statically indeterminable system (example 25.1, Fig. 34) we will obtain, if we take the constants of the exponential law:

$$\varepsilon'_e = 8{,}32 \cdot 10^{-10} \; \overset{(1)}{сек^{-1}}, \qquad \varepsilon''_e = 2{,}19 \cdot 10^{-5} \; \overset{(1)}{сек^{-1}},$$
$$\sigma'_e = 1{,}58 \; \overset{(2)}{\kappa\Gamma/\text{мм}^2}, \qquad \sigma''_e = 1{,}25 \; \overset{(2)}{\kappa\Gamma/\text{мм}^2},$$
$$\frac{\sigma_1}{1{,}58} - \frac{\sigma_2}{1{,}25} = 10{,}86,$$
$$2\sigma_1 + \sigma_2 = 48,$$

whence

$$\sigma_1 = 22{,}1 \; \overset{(2)}{\kappa\Gamma/\text{мм}^2},$$
$$\sigma_2 = 3{,}80 \; \overset{(2)}{\kappa\Gamma/\text{мм}^2},$$

Key: (1). s^{-1}. (2). kg/mm^2.

which in limits of accuracy of approximation coincides with the

obtained previously stressed state.

The exponential law of creep is conveniently used in tasks with variable (on coordinates or on time) temperature, since the approximations of the temperature dependences of the constants of exponential law are frequently proved to be simple. For some materials these approximations are given in the second part. One should only bear in mind that, utilizing these approximations, we somewhat lose in accuracy/precision in comparison with the use not of mediocre/satisfactory experimental data.

Example 31.1. Material of the rods of the farm/truss, depicted in Fig. 52, VT-14, the area of their cross section F=50 mm², l=200 mm, by α=45°. To determine peak load at temperature T=700°C, if the allowable speed of the displacement/movement of node O in steady state is 0.025 mm/s.

Equation of the equilibrium

$$\sigma_1 + \sqrt{2}\sigma_2 = \sigma, \qquad \sigma = \frac{P}{F}.$$

Condition of the consistency of the deformations

$$\dot{e}_1 = \frac{\dot{e}_2}{\cos^2\alpha} = 2\dot{e}_2 = [\dot{e}] = \frac{0{,}025}{200} = 1{,}25 \cdot 10^{-4} \ \mathrm{s}^{-1}.$$

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Consequently,

$$\varepsilon_e \exp\left(\frac{\sigma_1}{\sigma_e}\right) = 2\varepsilon_e \exp\left(\frac{\sigma_2}{\sigma_e}\right) = [\dot{e}],$$

$$\sigma_1 = \sigma_e \ln \frac{[\dot{e}]}{\varepsilon_e}, \quad \sigma_2 = \sigma_e \ln \frac{[\dot{e}]}{2\varepsilon_e},$$

$$\sigma = \sigma_e\left[\ln \frac{[\dot{e}]}{\varepsilon_e} + \sqrt{2}\ln \frac{[\dot{e}]}{2\varepsilon_e}\right] =$$

$$= \sigma_e\left[(1+\sqrt{2})\ln \frac{[\dot{e}]}{\varepsilon_e} - \ln 2\right] = 22{,}5 \text{ kg/mm}^2$$

and the permissible load $P=\sigma F=1125$ of kgf.

Example 31.2. Long shell of the alloy EP-202 is loaded by the internal pressure q=16 AT. Bore diameter of shell D=300 mm, the thickness of wall δ=2 mm. Construction/design is unloaded from axial forces. To determine maximum operating temperature, if the permissible increase in the diameter composes 2c/o after 1000 s of work.

We will use the approximation:

$$\varepsilon_e = \varepsilon_{e1} \exp\left(\frac{\beta\sigma}{T_0 - T} + \nu T\right). \qquad (*)$$

Values of constants in the range of temperatures of 800-1016°C:

$$\varepsilon_{e1} = 8{,}32 \cdot 10^{-18}\ ^{(1)}сек^{-1}, \quad \beta = 57{,}8\ ^{(2)}мм^2\ град/кГ,$$
$$T_0 = 1090°, \quad \nu = 0{,}0260\ ^{(3)}град^{-1}.$$

Key: (1). s^{-1}. (2). mm^2 deg/kgf. (3). deg^{-1}.

Voltage in duct σ=q D/2δ=12 kg/mm².

After designating $\mu = \ln\frac{\varepsilon}{\varepsilon_{e1}} = \ln\frac{2 \cdot 10^{-5}}{8{,}32 \cdot 10^{-18}} = 28{,}4$, let us record equation (*) in the form

$$T^2_{max} - \left(T_0 + \frac{\mu}{\nu}\right)T_{max} - \frac{\beta\sigma}{\nu} + \frac{T_0\mu}{\nu} = 0.$$

Solution of this equation at the extracted above values of the constants

$$T_{max} = 928°\ C.$$

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§32. Calculation statically indeterminable systems according to isochronal curves.

If the characteristics of creep are assign/prescribed in the form of isochronal curves, then the calculation of the stressed and state of strain of system at the moment of time t' after the application/appendix of constant external forces is reduced to solving of the task of plasticity, only for the curves of instantaneous deformation one should accept the isochronal curves, which correspond to the torque/moment of time t'.

In the process of the loading of the system of the voltage in

some "instantly" plastically deformed cell/elements, they can reverse the sign; in the examined above examples this case is encountered sufficiently frequently. There is no reliable experimental data on the creep of the material during the reversal of voltage at present; therefore it is necessary as the first approximation to use in such cases initial isochronal curves. This diagram is formally ne-contradictory during deformations $e > e_T$, which easily is perceived from Fig. 57, where for the case of linear strengthening (and of the absence of the similarity of isochronal curves) by solid lines is noted the behavior of material during the deformation of one sign and dash - during the reversal of the voltage in the range of instantaneous plastic deformation.

Example 32.1. To determine the elongation per unit length of construction/design in the example of 25.3 after t'=1000 s of work.

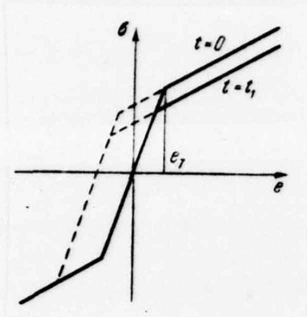

Fig. 57. Effect of the reversal of load on isochronal curves.

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The equations of equilibrium and consistency of deformations will be:

$$\sigma_1 + \frac{1}{2}\sigma_2 = \sigma,$$
$$e_1(\sigma_1) + \theta_1 = e_2(\sigma_2),$$

i.e.

$$\sigma_1 + \frac{1}{2}\sigma_2 = -5,$$
$$e_1(\sigma_1) + 1{,}04 \cdot 10^{-2} = e_2(\sigma_2). \qquad (*)$$

As function $\sigma_1(e_1)$ one should utilize $\sigma = \varphi(e)\psi(t')$, where $\phi(e)$ - the equation of the curve of instantaneous deformation and

$$\psi(t') = \frac{1}{1 + a \cdot (t')^{\beta}} = \frac{1}{1 + 0{,}1097 \cdot 1000^{0,3}} = 0{,}535.$$

Dependence $\sigma_1(e)$ is reproduced in Fig. 58. Further - the common procedure of the graphical solution of system (*) - table and Fig. 59.

Example 32.2. The two-layered thin-walled tube, depicted in Fig. 60, is made made of copper M1 (internal wall) and stainless steel Kh18N10T (external).

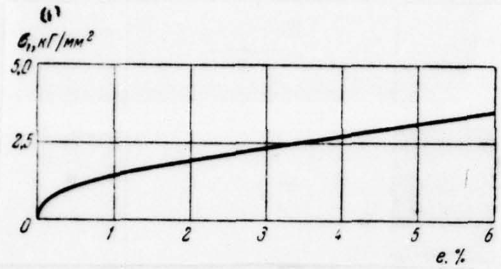

Fig. 58. To example 32.1. dependence σ_1 (e).

Key: (1). kg/mm².

(1) σ_2 кГ/мм²	$e_2 \cdot 10^2$	$e_1 \cdot 10^2$	(1) σ_1 кГ/мм²
−5,0	−1,22	−2,26	−2,00
−5,5	−1,48	−2,52	−2,12
−6,0	−1,74	−2,78	−2,25

σ^*_1=-2,165 kg/mm², σ^*_2=-5,67 kg/mm², e_{t-1000} =e_2=-1,56o/o.

Key: (1). kg/mm².

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Duct is loaded by internal pressure, operating temperature of construction/design T=800°. To determine the permissible pressure q, if the permissible deformation of duct after 1200 s - 5o/o. From the equation of the equilibrium

$$q = 2\frac{\sigma_1\delta_1 + \sigma_2\delta_2}{D}.$$

Since the coefficients cf the linear expansicn of copper and stainless steel are close, then it is possitle to disregard temperature terms in the equation cf the ccnsistency of the deformation

$$[e] = e_1(\sigma_1) = e_2(\sigma_2) = 5 \cdot 10^{-2}.$$

On isochronal curved for the stainless steel for t=1200 s let us determine σ_2=15,7 kg/mm². For copper

$$\psi(1200) = \frac{1}{1 + at^{\beta}} = \frac{1}{1 + 0{,}239 \cdot 1200^{0,3}} = 0{,}339$$

and

$$\sigma_1 = 1{,}185 \text{ kg/mm}^2.$$

Consequently, q=2(1.185•5+15.7•1)/200=0.216 kgf/mm²=21.6 at.

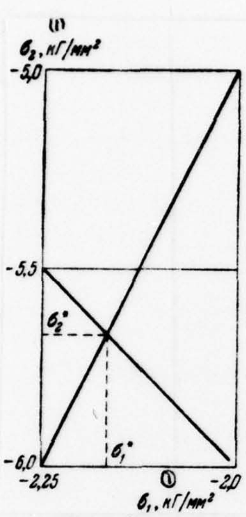

Fig. 59.

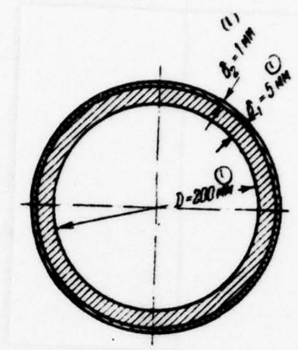

Fig. 60.

Fig. 59. To example 32.1. Graphic determination of voltages σ^*_1 and σ^*_2.

Key: (1). kg/mm².

Fig. 60. Diagram of given one in example 32.2 two-layered thin-walled tubes.

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Chapter IV.

BEND.

§33. Determination of instantaneous defcrmaticn with the pure bending of the crux of symmetrical airfcil/profile.

Let us accept for x and y axes of the axis of the symmetry of the airfoil/profile of section/cut, it is directed z axis along the axis of rod. Let the bending pair be applied in plane yOz. Since the instantaneous diagram fcr elongaticn and ccmpression is identical, the axis of symmetry Ox will be the axis of the symmetry of section/cut (Fig. 61).

Accepting the **hypothesis** cf flat/plane section/cuts, let us assume

$$e_0 = ky.$$

Here k - curvature of **bent axle of** rod. Consequently,

$$\sigma = \varphi(ky).$$

Function $\phi(e_0)$ is inversion of a function $e_0(\sigma)$; this function is assign/prescribed instantaneous stress-strain diagram. As a result of

the odd parity of function $\phi(e_0)$ and the symmetry of section/cut, the condition of equality tc zero axial force is satisfied automatically. The bending moment is lccated as fcllows:

$$M=2\int_0^h \sigma\, by\, dy. \qquad (33.1)$$

Here h - half the height of section/cut. During the computation of this integral, it is convenient to accept for the variable of integration value $\xi=ky$, the width of secticn/cut b exists function $y/h=ky/kh=\xi/\xi_0$, $\xi_0=kh$, therefore,

$$M=\frac{2h^2}{\xi_0^2}\int_0^{\xi_0}\varphi(\xi)\, b\left(\frac{\xi}{\xi_0}\right)\xi\, d\xi.$$

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Let us assume

$$\psi(\xi_0)=\frac{2}{b_0\xi_0^2}\int_0^{\xi_0}\frac{\varphi(\xi)}{\sigma_*}\,b\left(\frac{\xi}{\xi_0}\right)\xi\,d\xi. \qquad (33.2)$$

Here b_0 - characteristic width of section/cut, σ_* - any constant, which has the dimensionality of voltage. Then $\psi(\xi_0)$ - the dimensionless function of the dimensionless parameter ξ_0. Now we can write

$$M=b_0h^2\sigma_*\psi(kh). \qquad (33.3)$$

Formula (33.3) establish/installs the dependence between curvature and the bending moment, function $\psi(kh)$ is easily located by the graphical integration.

Let us assume now, that the temperature over the section/cut of rod is distributed unevenly, therefore, T=T(y). In this case, let us for the moment consider that T(y)=T(-y), stress distribution remains symmetrical relative to axle/axis Ox. Equation (33.1) will take the new following form:

$$M=2\int_0^h\varphi(ky-\theta)\,b(y)\,y\,dy. \qquad (33.4)$$

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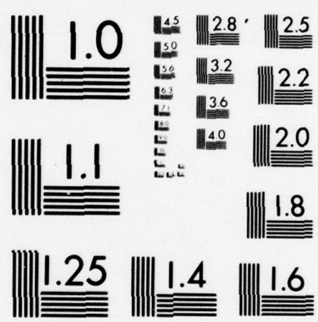

MICROCOPY RESOLUTION TEST CHART
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Here $\theta(y)=\alpha T(y)$ - thermal deformation.

Let be assign/prescribed the series of the curves of instantaneous deformation for temperatures $T_1, T_2, \ldots, T_m$. For the half section/cut, it is constructed the diagram/curve of the temperature distribution on height. Let us note on this temperature distribution $T_1, T_2, \ldots, T_m$, it is measured the ordinate of the points of the section/cuts in which the temperature accepts each of these values. Thus, to temperature T_i corresponds ordinate y_i and the width of section/cut b_i. Value θ_i at point y_i is known, if is known the temperature: $\theta_i = \alpha T_i$.

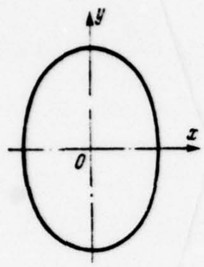

Fig. 61. Diagram of symmetrical airfoil/profile.

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Now we are assigned by the arbitrary value of k and compute the values of integrand in formula (33.4) for $y=y_i$. In this case, $\varphi(ky_i-\theta_i)$ it is located directly as ordinate of the curve of deformation for the appropriate temperature, $b=b(y_i)$ is taken from drawing. After applying on the graph of the points, which represent the values of integrand in function y and after connecting them to steady curved, we find the limited by it area. Computations are repeated for the different values of k.

On the diagram of instantaneous deformation, usually there are two sections, linear and nonlinear. For a linear section integral (33.2) is calculated easily; in this case, it is necessary to assume $\phi=E\xi$. Only for the values ξ_0, which exceed the deformation of limit

cf proportionality, it is necessary to manufacture numerical integration. On the other hand, there are materials (copper, for example), whose diagram of instantaneous deformation does not have the distinguishable linear section and on an entire extent is approximated by exponential function. For this approximation integral (33.2) also can be easily calculated for many cross sections in the closed form. Are given below some examples of calculations for each cases.

Example 33.1. Rod is made from the alloy D16AT, section/cut rectangular, the height of section/cut 2h, width b_0, temperature of 300°C. For rectangular cross section throughout formula (33.2) we have

$$\psi(\xi_0)=\frac{2}{\xi_0^2}\int_0^{\xi_0}\varphi(\xi)\,\xi\,d\xi.$$

The curve of instantaneous deformation is given to Fig. 95, it has a linear section $0<e<0,27\cdot10^{-2}$, a value of the module/modulus of elasticity $E=3,8\cdot10^3$ kg/mm².

In elastic range $\varphi=E\xi$, $\psi(\xi_0)=\frac{2}{3}E\xi_0$, consequently,

$$\psi(kh)=\frac{M}{b_0h^2\sigma_*}=\frac{2}{3}Ekh=2,96\cdot10^3kh,\qquad \sigma_*=1 \text{ kg/mm}^2.$$

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This relationship/ratio is correct, thus far value kh does not exceed the deformation of limit of proportionality $0.27 \cdot 10^{-2}$ and $\psi \leqslant 6{,}83$ kg/mm². With $kh=\xi_0>0.27 \cdot 10^{-2}$ function $\psi(\xi_0)$ is calculated by numerical integration, results are given in the following table.

$\xi \cdot 10^2$	0,27	0,35	0,40	0,45	0,50	0,60	0,70	0,80	0,90
(1) $\varphi, \frac{\kappa\Gamma}{\text{мм}^2}$	10,1	12,3	13,4	14,3	14,8	15,7	16,3	16,9	17,3
(1) $\psi, \frac{\kappa\Gamma}{\text{мм}^2}$	6,83	8,68	9,80	10,68	11,40	12,60	13,50	14,20	14,82

Key: (1). kg/mm².

Value ϕ are stress levels, measured according to stress-strain diagram, function ψ was calculated by integration for the rule of trapezoids. Figures 62 gives the graph/diagram of dependence ψ on $\xi_0=kh$.

Example 33.2. Let us assume that is known function ψ for certain solid section, for example limited by external outline/contour in Fig. 63. Let us examine now hollow section, such, that its internal outline/contour is such/similar to external with the factor of similarity γ. Let us designate $\psi(\xi_0,\gamma)$ the function of flexure for gently section/cut. Calculating integral in formula

(33.2), we can first find the integral, distributed by an entire area, limited by external outline/contour, and then deduct the same integral by the area, limited by internal outline/contour. This second integral will take accurately the same form as the first, only value b in it must be replaced by γb and value ξ_0 by $\gamma\xi_0$.

Thus,

$$\psi(\xi_0, \gamma) = \\ = \frac{2}{\sigma_* b_0 \xi_0^2}\left[\int_0^{\xi_0} \varphi(\xi)\, b\left(\frac{\xi}{\xi_0}\right)\xi\, d\xi - \gamma\int_0^{\gamma\xi_0} \varphi(\xi)\, b\left(\frac{\xi}{\xi_0}\right)\xi\, d\xi\right].$$

Hence

$$\psi(\xi_0, \gamma) = \psi(\xi_0, 0) - \gamma^3\psi(\gamma\xi_0, 0). \qquad (33.5)$$

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Here $\psi(\xi_0, 0)$ - the function of flexure for solid section.

Formula (33.5) makes it possible, for example, to calculate for flexure the rods of tubular section/cut, if is known the dependence of torque/moment on curvature for the rod of solid circular section/cut.

Example 33.3. It is required to determine the dependence of torque/moment on curvature for the solid rod of the round cross-section, manufactured from copper M-1.

As is known, of copper the elastic section of the curve of instantaneous deformation is not separated, the dependence of deformation on voltage is approximated well by the exponential function

$$\sigma = Ke^{m}.$$

Let us calculate function ψ for the solid round cross-section:

$$\int_0^{\xi_0} K\xi^{m+1}\left[1-\left(\frac{\xi}{\xi_0}\right)^2\right]^{1/2} d\xi = K\xi_0^{m+2}\frac{2^m}{3+m}\frac{\left[\Gamma\left(1+\frac{m}{2}\right)\right]^2}{\Gamma(m+2)}.$$

Here Γ - the tabulated function (for example, see E. Yanke, F. Emde, the table of functions with formulas and curves, Fizmatgiz, 1959).

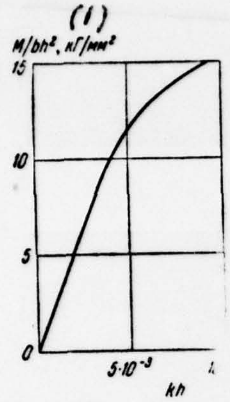

Fig. 62.

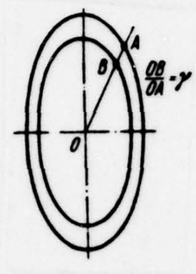

Fig. 63.

Fig. 62. To example 33.1. Dependence of the bending moment on curvature.

Key: (1). kg/mm².

Fig. 63. Diagram of assigned/prescribed to example 33.2 symmetrical airfoil/profiles it is gently rod.

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It is represented now expression for ψ in the following form:

$$\psi(\xi_0, 0) = \chi(m) K \xi_0^m = \chi(m) \varphi(\xi_0),$$

$$\chi(m) = \frac{2^{1+m}}{3+m} \frac{\left[\Gamma\left(1+\frac{m}{2}\right)\right]^2}{\Gamma(m+2)}.$$

Convenience in this formula lies in the fact that the plotted function ψ is such/similar to the diagram of deformation. In the

following table are given the values of the factor of similarity $\chi(m)$ for copper at different temperatures, in this case, they were utilized of the data of Fig. 130.

T° C	m	$\chi(m)$
20	0,525	0,988
400	0,485	1,008
600	0,457	1,015
800	0,403	1,051

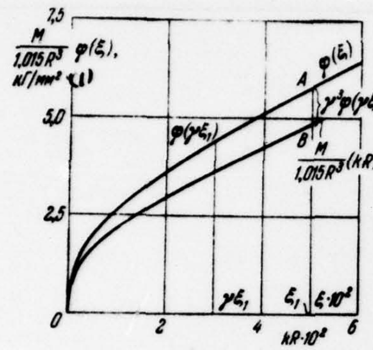

Fig. 64. To the determination of the dependence of the bending moment on curvature in example 33.4.

Key: (1). kg/mm².

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Example 33.4. It is required to determine the dependence of torque/moment on curvature for a tube from copper M-1 with an outside radius of R and the ratio of an inside radius to external γ=0,6 at temperature of 600°.

In accordance with formula (33.3) and the results of examples of 33.2 and 33.3 this dependence is given by the expression

$$M = R^3\chi(m)[\varphi(kR) - \gamma^3\varphi(\gamma kR)] = \\ = 1{,}015R^3[\varphi(kR) - 0{,}6^3\varphi(0{,}6kR)].$$

Figures 64 gives the curve of instantaneous deformation for copper at 600°. The plotting of curves of the dependence of torque/moment on curvature is manufactured in a following manner. We take certain point A of the curve of instantaneous deformation with abscissa ξ_1, its ordinate $\phi(\xi_1)$, is measured the ordinate of point with abscissa $\gamma\xi_1$, i.e., $\phi(\gamma\xi_1)$, it is multiplying the obtained value on γ^3 and we set aside cut $AB=\gamma^3\phi(\gamma\xi_1)$ from point A on vertical line downward. Point B belongs to unknown curved, that gives dependence $M/1{,}015R^3$ on kR for a tube.

§34. Determination of instantaneous deformation with pure bending in the plane of symmetry.

Let us assume now that the rod has only one plane of symmetry, namely plane yOz, in which is applied the bending pair. Let us consider also that the temperature is the function of coordinate y, but it does not depend on coordinate x. If section/cut is symmetrical relative to axle/axis Ox, but temperature is distributed by asymmetric form, so that $T(y)\neq T(-y)$, the investigation of flexure is necessary to manufacture according to the method, set-forth in this paragraph. Essential complication in comparison with the preceding/previous task will consist in the fact that the position of

the neutral axle/axis of flexure is here previously unknown. If y axis exists an axis of the symmetry of the airfcil/profile of section/cut, then for x axis we must accept neutral axle/axis, then as before e=ky and $\sigma=\Phi(ky)$. But in this case must be carried out the condition cf equality to zero axial force, namely

$$\int_F \sigma\, dF = \int_F \varphi(ky)\, dF = 0. \qquad (34.1)$$

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In the general case frcm condition (34.1) it follows that the position of neutral axle/axis depends on curvature k. This means that with an increase in the bending moment the neutral axle/axis is moved. The relationship/ratio between the bending moment and curvature there will be following:

$$M = \int_{-h_1}^{h_2} \sigma\, by\, dy. \qquad (34.2)$$

Here h_1 and h_2 - distance from neutral axle/axis to the extreme points of section/cut snizu and on top respectively. Precise analysis of elasto-plastic flexure taking into account the true position of neutral axle/axis is connected with sufficiently cumbersome calculations. At the same time dependence (34.2) proves to be sufficiently little that is sensitive to the small errors in the definition of the position of neutral axle/axis; therefore, it is possible to recommend the approximate procedure, which consists of

the fact that the neutral axle/axis is defined in the manner that if the behavior of material with flexure was elastic. In this case, it is necessary only to consider with the fact that the modulus of elasticity depends on temperature. Thus, let us assume that

$$\sigma = E(T)e.$$

Let us conduct the arbitrary auxiliary axle/axis $O\xi$, perpendicular to axle/axis Oy (Fig. 65), unknown distance from it to neutral axle/axis let us designate η_0, the distance of the arbitrary point of section/cut from axle/axis ξ let us designate η. From drawing it is evident that

$$y = \eta - \eta_0.$$

Now condition (34.1) is led to following:

$$\int_F (\eta - \eta_0) E\, dF = 0.$$

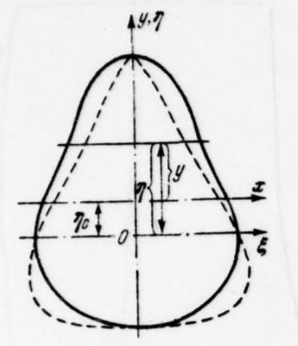

Fig. 65. Diagram of the airfoil/prcfile, which has one axis of symmetry.

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Hence

$$\eta_0=\frac{\int\limits_F \frac{E}{E_0}\eta\, dF}{\int\limits_F \frac{E}{E_0}\, dF}=\frac{S_0}{F_0}. \qquad (34.3)$$

The numerator of the right side of formula (34.3) is the area moment ratio of cross section with a weight of E/E_0, denominator - cross-sectional area with the same weight. Value E_0 - arbitrary constant, for example the modulus of elasticity when $\eta = 0$. For computing values S_0 and F_0, we enter as follows. We replace the cross section of rod with the figure, depicted by primes, which is constructed as follows. The width of section/cut at a distance η

from axle/axis ξ, $b=b(\eta)$ is changed in ratio E/E_0, where E - the modulus of elasticity, which corresponds to the temperature in point with coordinate η. Numerator and denominator in formula (34.3) are static moment with respect to axle/axis $O\xi$ and the area of the modified section/cut.

After the position of neutral axle/axis it is found, bending computation is manufactured in exactly the same way just as for a rod with two axes of symmetry. It is certain, instead of formula (34.3) now it is necessary to use the following formula, in which as x axis is accepted the neutral axle/axis:

$$M=\int_{-h_1}^{h_2}\varphi(ky-\theta+\theta_0)\,b(y)\,y\,dy, \qquad (34.4)$$

where

$$\theta=\alpha T(y), \qquad \theta_0=\alpha T(0).$$

In appendices frequently instantaneous deformation can be considered elastic. Then from (34.4) it follows

$$k\int Ey^2\,dF-\int Ey\,\Delta\theta\,dF=M, \quad \Delta\theta=\theta-\theta_0.$$

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Value

$$J_0=\int\frac{E}{E_0}y^2\,dF$$

is the area moment of inertia of the modified section/cut relative to axle/axis Ox. Let us introduce another designation:

$$k_T=-\frac{1}{J_0}\int\frac{E}{E_0}\Delta\theta y\,dF.$$

Then communication/connection between torque/moment and curvature there will be following:

$$k = \frac{M}{E_0 J_0} + k_T. \qquad (34.5)$$

Example 34.1. **To determine the dependence between** the bending moment and curvature for the crux of t-section PR 109 No. 3. Material is alloy D16AT, temperature of 300°.

Figure **66** gives the **section/cut, schematized** for calculation. The value of the moment of **inertia and the** center-of-gravity location are undertaken from handbook, $J_x = 14\,420$ mm^4, h_2=8.44 mm. With T=300° E=3.8•10^3 kg/mm^2, the deformation of limit of proportionality e'=0.27•10^{-2}.

In the elastic range

$$M = kEJ_x = k3{,}8 \cdot 10^3 \cdot 14\,420 = k5{,}48 \cdot 10^7 \text{ kgf•mm.}$$

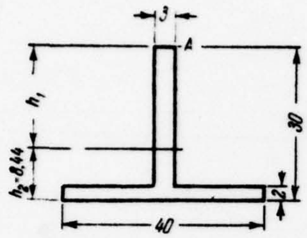

Fig. 66.

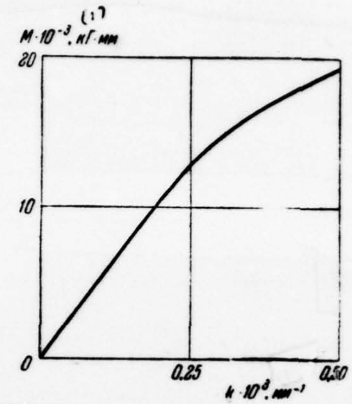

Fig. 67.

Fig. 66. Diagram of given one in an example 34.1 airfoil/profiles.

Fig. 67. To example 34.1. Dependence of the bending moment on curvature.

Key: (1). kgf/mm.

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y мм	by мм²	Δy мм	$ky \cdot 10^2$	σ кГ/мм²	σby кГ	$\frac{(\sigma by)_i + (\sigma by)_{i+1}}{2}$ кГ	$\frac{(\sigma by)_i + (\sigma by)_{i+1}}{2} \Delta y$ кГ·мм
−8,44	−337		−0,211	−7,88	2650		
		0,5				2500	1250
−7,94	−318		−0,198	−7,38	2350		
		0,5				2210	1105
−7,44	−298		−0,186	−6,94	2070		
		0,5				1930	965
−6,94	−278		−0,173	−6,46	1790		
		0,5				1670	835
−6,44	−258		−0,161	−6,01	1550		
		0				0	0
−6,44	−19,3		−0,161	−6,01	116		
		2,0				85	170
−4,44	−13,3		−0,111	−4,14	55		
		4,44				28	124
0	0		0	0	0		
		5				35	175
5	15		0,125	4,67	70		
		5				175	875
10	30		0,250	9,34	280		
		5				430	2150
15	45		0,375	12,90	580		
		5				735	3670
20	60		0,500	14,85	890		
		1,56				938	1460
21,56	64,6		0,544	15,25	985		
							$M = 12\ 780$ кГ·мм

[кГ = kg]

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Plastic deformation first of all appears in point A, with this

$$k = k' = \frac{e'}{h_1} = \frac{0{,}27 \cdot 10^{-2}}{21{,}96} = 0{,}125 \cdot 10^{-3}\ мм^{-1},$$
$$M = 6620\ кГ \cdot мм. \quad (1)$$

Key: (1). kg·mm.

Computation M for a plastic region at value $k=0{,}25 \cdot 10^{-3}$ is given in table. Analogous computations for

the series of values k give the dependence of torque/moment on curvature, given to Fig. 67.

Example 34.2. The rod of rectangular cross section 10x20 mm is made made of steel EI-654 and is bent in the plane of the greatest rigidity. Temperature is changed according to linear law on the height of section/cut, its distribution according to width is evenly, on the upper plane T=900°, on lower that T=700°. The initial temperature of 20°. It is required to find the dependence of the bending moment on curvature and to plot diagram/curves voltage.

For axle/axes ξ and η let us accept the axes of the symmetry of section/cut. In Fig. 68 are constructed the following diagram/curves: a) the temperature distribution on height, b) the dependence of the modulus of elasticity on height, c) the dependence of the given width bE/E_0 on height.

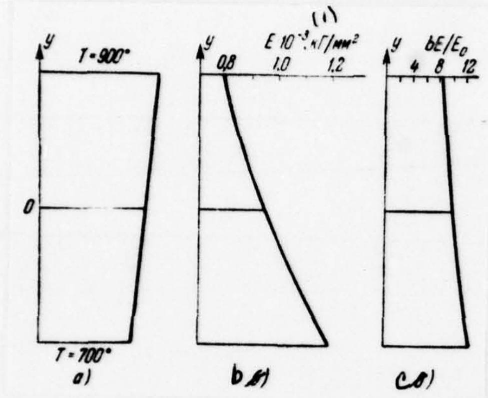

Fig. 68. To example 34.2. Diagram/curves T, E and E/E_0.

Key: (1). kg/mm².

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In this case, for value E_0, was accepted value E when $\eta = 0$. The diagram/curve of the given width it was constructed on millimeter graph paper, the calculated by the graphical integration cross-sectional area render/showed F_0=205 mm². Static moment S_0 is calculated by numerical integration for simpson's rule, is obtained S_0=-144 mm³; therefore

$$\eta_0 = \frac{S_0}{F_0} = -0,7 \text{ мм}.$$

We are now assigned by the consecutive values of k and compute M for formula (34.4) by numerical integration. In this case, the values

of function $\phi(ky+\theta-\theta_0)$ are taken from curved instantaneous deformation at the appropriate temperatures. The results of computations are given in the following table.

$k \cdot 10^3$, мм$^{-1}$	−1	−0,5	−0,25	0	0,5	1
$M \cdot 10^{-3}$, кГ · мм (1)	−19,4	−14,45	−4,90	10,45	18,32	21,00

Key: (1). kg •mm.

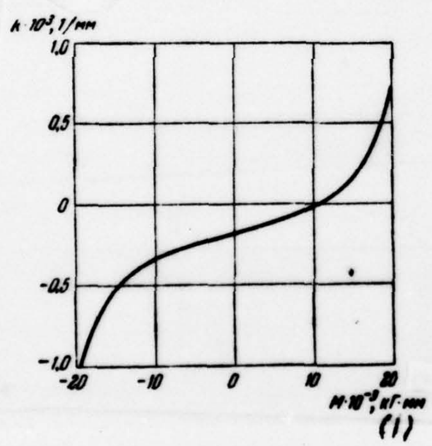

Fig. 69. To example 34.2. Dependence of k on M.

Key: (1). kgf•mm.

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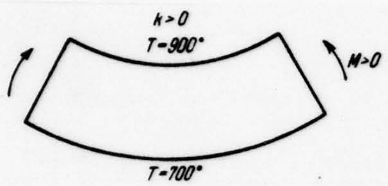
Figure 69 depicts the graph/diagram of dependence of k on M; the diagram, given to Fig. 70, explains the rule of signs. If M=0, as a result of uneven heating rod obtains the negative curvature, equal to $1.7 \cdot 10^{-4}$ mm^{-1} (point of intersection curved with the axle/axis of abscissas). In order to hold rod in the undistorted state, to it it is necessary to apply the positive torque/moment, equal to $10.45 \cdot 10^{3}$ kgf•mm (point of intersection curved with the axle/axis of abscissas).

The diagrams of voltages are constructed on the formula

they are given to Fig. 71.



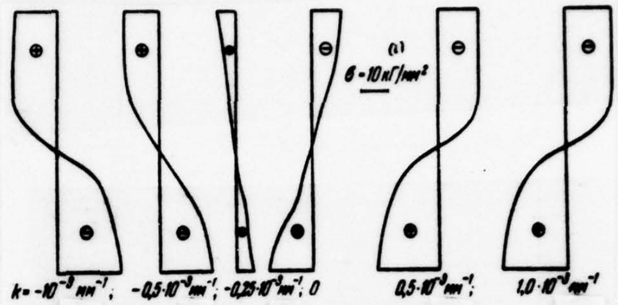


Fig. 70. The diagram of the bent beam, elucidating is the rule of signs.

Fig. 71. To example 34.2. Diagram/curves σ fcr different k.

Key: (1). kg/mm².

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§35. Instantaneous bending strain cf the cruxes of curved profile.

Let us assume now that the section/cut cf rod dces not have axes of symmetry and temperature is distributed over section/cut by arbitrary form. Solving of the task of elastc-plastic flexure under these conditions is proved to be complex; therefore we will be restricted to the examination of the elastic deformation of this rod, by taking into account the dependence of the modulus cf elasticity on

temperature. This procedure is justified by the fact that, as a rule, instantaneous plastic deformations were small. Let us relate section/cut to arbitrary axle/axes $\xi O \eta$ (Fig. 72), let us calculate changed (with weight E/E_0) area, static moments, axial and products of inertia on the formulas

$$\left.\begin{aligned} F&=\int \frac{E}{E_0}dF, & S_\xi&=\int \eta\frac{E}{E_0}dF, & S_\eta&=\int \xi\frac{E}{E_0}dF, \\ J_\xi&=\int \eta^2\frac{E}{E_0}dF, & J_\eta&=\int \xi^2\frac{E}{E_0}dF, & J_{\xi\eta}&=\int \xi\eta\frac{E}{E_0}dF. \end{aligned}\right\} \tag{35.1}$$

During the computation of the integrals, which figure as in formulas (35.1), one should to break area into cell/elements, to note the coordinates of the center of gravity of each cell/element ξ_k, η_k, to find value $E_k = E(\xi_k, \eta_k)$ and replace integrals with the appropriate sums. Further course of computation in no way differs from analogous calculation for a material with the constant modulus of elasticity. The coordinates of the center of gravity of section/cut are located through the formulas

$$\xi_0 = \frac{S_\eta}{F}, \qquad \eta_0 = \frac{S_\xi}{F}.$$

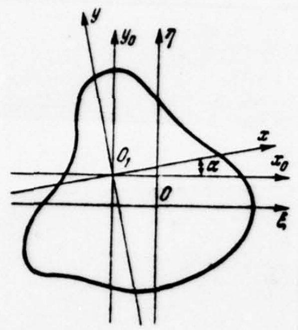

Fig. 72. Diagram of curved profile.

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Point O_1 with these coordinates is the center of gravity of the figure of alternating/variable density, equal to E/E_0. Now are conducted axle/axes x_0, y_0 with beginning in the center of gravity O_1, parallel to axle/axes ξ, η, are calculated torque/moments of inertia relative to these axle/axes

$$J_{x_0} = J_\xi - F\eta_0^2, \quad J_{y_0} = J_\eta - F\xi_0^2, \quad J_{x_0y_0} = J_{\xi\eta} - F\xi_0\eta_0.$$

The principal axes of inertia of section/cut are turned relative to axle/axes x_0, y_0 to angle α, in this case

$$\operatorname{tg} 2\alpha = \frac{2J_{x_0y_0}}{J_{x_0} - J_{y_0}}.$$

Principal moments of inertia are located now through the known formulas:

$$J_x = \frac{1}{2}\left[(J_{y_0} + J_{x_0}) \pm \sqrt{(J_{y_0} - J_{x_0})^2 + 4J_{x_0y_0}^2}\right],$$
$$J_y = \frac{1}{2}\left[(J_{y_0} + J_{x_0}) \mp \sqrt{(J_{y_0} - J_{x_0})^2 + 4J_{x_0y_0}^2}\right].$$

Now as a result of the law cf the flat/plane section/cuts

$$e = k_x y - k_y x + e_0$$

and

$$\sigma = E(k_x y - k_y x + e_0 - \theta).$$

The equations of equilibrium will be fcllowing:

$$M_x = \int \sigma y\, dF, \quad M_y = \int \sigma x\, dF, \quad N = \int \sigma\, dF = 0.$$

Taking into account the introduced previously designations, we will cbtain

$$e_0 = \frac{1}{E_0 F} \int E\theta\, dF,$$
$$k_x = \frac{M_x}{E_0 J_x} + \frac{1}{E_0 J_x} \int E\theta y\, dF, \; k_y = \frac{M_y}{E_0 J_y} - \frac{1}{E_0 J_y} \int E\theta x\, dF. \quad (35.2)$$

Now

$$\sigma = \frac{E}{E_0}\left[\frac{M_x y}{J_x} - \frac{M_y x}{J_y} + y \int E\theta y\, dF - \right.$$
$$\left. - x \int E\theta x\, dF + \frac{1}{F} \int E\theta\, dF - E_0\theta\right].$$

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§36. The steady-state creep of the evenly heated rod whose section/cut has two axes of symmetry.

This case is the simple; for the specific laws of creep and

forms of cross section solution it is located in the closed form. Let the section/cut take the form, depicted in Fig. 61. As a result of the law of the flat/plane section/cuts

$$\varepsilon = \varkappa y.$$

Here $\varkappa^{\cdot}$ - rate of change in the curvature with flexure, $\varkappa = \dot{k}$. As a result of the accepted law of creep

$$\varkappa y = v(\sigma), \qquad \sigma = s(\varkappa y).$$

Here $s(\varepsilon)$ - the function, inverse functions $v(\sigma)$.

Compiling an equation of equilibrium, we will obtain

$$M = 2\int_0^h s(\varkappa y)\, b\, dy. \tag{36.1}$$

Let us examine now special cases.

a). Power law. Let us assume

$$\varkappa_n = \frac{\varepsilon_n}{h}, \quad M_n = 2\sigma_n \frac{1}{h^{1/n}} \int_0^h b(y)\, y^{1+\frac{1}{n}}\, dy. \tag{36.2}$$

Then communication/connection between rate of change in the curvature and the bending moment there will be following:

$$\varkappa = \varkappa_n \left(\frac{M}{M_n}\right)^n. \tag{36.3}$$

For rectangular cross section with a width of b by height 2h we find

$$M_n = \frac{2n}{2n+1}\sigma_n b h^2.$$

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In table gives corrected values on n of the factor depending in expression M_n:

n	1	2	3	4	5	6	7	8	10	∞
$\frac{2n}{2n+1}$	0,667	0,800	0,858	0,889	0,908	0,923	0,934	0,942	0,953	1

Case n=∞ corresponds to the distributicn cf voltages over section/cut, the same as the stress distribution in perfectly plastic rod.

For the circle/wheel of radius R

$$M_n = \frac{2^{2+\frac{1}{n}}}{3+\frac{1}{n}} \frac{\left[\Gamma\left(1+\frac{1}{2n}\right)\right]^2}{\Gamma\left(2+\frac{1}{n}\right)} R^3\sigma_n.$$

Table gives corrected values of value $\mu = M_n/R^3\sigma_n$ in dependence on n.

n	1	2	3	4	5	6	7	8	10	∞
μ	0,785	0,998	1,091	1,146	1,180	1,204	1,221	1,234	1,252	1,333

This same dependence is given to Fig. 73. For the ring of the outside radius R and of internal r

$$M_n = \frac{2^{2+\frac{1}{n}}}{3+\frac{1}{n}} \frac{\left[\Gamma\left(1+\frac{1}{2n}\right)\right]^2}{\Gamma\left(2+\frac{1}{n}\right)} R^3\sigma_n\left(1-\frac{r^3}{R^3}\right),$$

therefore in the calculations of round ducts it is possible to use the data of given table and Fig. 73, one should only assume

$$\mu = \frac{M_n}{R^3\sigma_n\left(1-\frac{r^3}{R^3}\right)}.$$

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b). Exponential law. Since

$$\sigma=\sigma_e\left|\ln\frac{\varkappa y}{\varepsilon_e}\right|\operatorname{sign} y=\sigma_e\left|\ln\frac{\varkappa h}{\varepsilon_e}+\ln\frac{y}{h}\right|\operatorname{sign} y,$$

that

$$M=2\sigma_e\left[\ln\frac{\varkappa h}{\varepsilon_e}\cdot\int_0^h by\,dy+\int_0^h by\ln\frac{y}{h}\,dy\right].$$

Let us assume

$$\frac{\int_0^h by\ln\frac{y}{h}\,dy}{\int_0^h by\,dy}=\ln\beta. \tag{36.4}$$

Then the relationship/ratio between rate of change in the curvature and torque/moment is led to the following form:

$$\varkappa=\varkappa_e\exp\left(\frac{M}{M_e}\right). \tag{36.5}$$

Here

$$\varkappa_e=\frac{\varepsilon_e}{\beta h};\qquad M_e=2\sigma_e\int_0^h by\,dy. \tag{36.6}$$

For the rectangle

$$\beta=e^{-1/2}=0{,}546,\qquad M_e=\sigma_e bh^2.$$

Let us note that in the task of flexure the exponential law of creep leads to the physically impossible law of the distribution of voltages over section/cut. It is real/actual, formula for voltages there will be following:

$$\sigma=\sigma_e\left\{\ln\left(\frac{y}{\beta h}\right)+\frac{M}{M_e}\right\}. \qquad (36.7)$$

With y=0, i.e., on neutral axle/axis, the voltage takes infinite value. However, the role of this infinite voltage in the creation of the bending moment is insignificant, if torque/moment is sufficiently great, and only under these conditions the application/use of an exponential law is justified.

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As concerns formula (36.7), it must be noted that the first term in the curly brace does not depend on applied moment; therefore during an increase in the torque/moment predominating becomes the second term, which gives the even distribution of voltages according to the section/cut of rod. Desiring to avoid the infinite values of voltages, we must either replace the exponential law of creep with the law of the hyperbolic sine (see §14), or to calculate, that the logarithmic function for voltages is accurate when $y>\gamma h=\frac{e_e}{\varkappa}$, whereas with $y<\gamma h$, $\sigma=0$. During the computation of integrals (36.4) and (36.6) it will be changed lower limit, as a result we will obtain

$$\ln\beta = -\frac{\frac{1}{2} + \gamma^2\left(\ln\gamma - \frac{1}{2}\right)}{1-\gamma^2},$$
$$M_e = \sigma_e bh^2(1-\gamma^2).$$

The obtained correction is insignificant, and we will not take it into consideration.

In practical calculations they use usually the constants of creep, determined in certain limited range of change in the voltages (or the rates of steady-state creep), therefore, solving the problems of flexure, we always, even in the case of power law, we allow/assume certain indeterminancy/uncertainty, connected with the need for the extrapolation of experimental dependence. It is obvious that, after manufacturing this extrapolation into the range of small voltages by different methods, we will not obtain the essential difference in dependence M - $\varkappa$. It is important only so that the law of creep would be reliably establish/installed for the range of the voltages, realized in the peripheral parts of the section/cut.

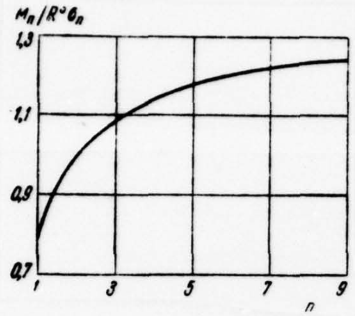

Fig. 73. Dependence μ on n for the crux of round section.

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§37. Case of variable temperature.

If the conditions of the preceding/previous task are complicated by that fact that the temperature is variable, but the temperature distribution is symmetrical relative to x, so that T(y)=T(-y), then course of computation it remains in principle previous and formula (36.1) retains force.

a). Power law. Constant ε_n always can be selected so that it would not depend on the temperature.

On formula (36.1)

$$M=2\int_0^h \sigma_n\left(\frac{xy}{\varepsilon_n}\right)^{1/n} by\,dy.$$

If index n is constant for that temperature range which is realized in rod, then hence again it follows formula (36.3), in which $\varkappa_n$ is determined by the as before first of relationship/ratios (36.2), whereas

$$M_n = \frac{2}{h^{1/n}} \int_0^h \sigma_n b(y)\, y^{1+\frac{1}{n}}\, dy. \qquad (37.1)$$

In the case of the variable n, it is necessary to be assigned by values $\varkappa$ and to numerically find value M for each $\varkappa$, simple dependence in this case is not obtained.

b). Exponential law. Let us assume as before

$$\sigma = \sigma_e \ln \frac{\varkappa y}{e_e},$$

but let us now consider that σ_e and e_e they are the functions of temperature, and consequently, as assigned/prescribed the functions y.

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It is represented this expression as follows:

$$\sigma = \sigma_e \left\{ \ln \frac{\varkappa h}{e_*} + \ln \frac{e_* y}{e_e h} \right\}.$$

Here e_* - any value, which has dimensionality e_e. Substituting in formula (36.1) and integrating, we again will arrive at

relationship/ratio (36.5), no in this case will be

$$\left.\begin{aligned} \varkappa_e = \frac{\varepsilon_*}{\beta h}, \qquad M_e = 2\int \sigma_e b y\, dy, \\ \ln\beta = \frac{\int \sigma_e b y \ln\left(\frac{\varepsilon_*}{\varepsilon_e}\frac{y}{h}\right) dy}{\int \sigma_e b y\, dy}. \end{aligned}\right\} \qquad (37.2)$$

Stress distribution is determined as before by formula (36.7).

§38. Flexure of the rod, which has one plane of symmetry.

Let us assume that the bending pair acts in the plane of symmetry. A basic question will now consist in the determination of the position of neutral axle/axis. A precise determination of the position of neutral axle/axis meets difficulty; therefore we will use the approximate procedure, which consists of the fact that the neutral axle/axis is defined just as for a rod from perfectly plastic material in limiting condition. If yield point is constant, which corresponds to the evenly heated rod, then neutral axle/axis divides cross-sectional area into two equivalent parts.

An error in this procedure can be estimated based on the example of section/cut in the form of isosceles triangle (Fig. 74). If n=1, then neutral axle/axis it passes through the center of gravity of section/cut, i.e., is located from apex/vertex on 2/3h (axle/axis x_1).

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If n=∞, then neutral axle/axis x_∞ divides cross-sectional area into two equivalent parts; therefore its distance frcm apex/vertex is equal to $\frac{1}{\sqrt{2}}h$. The distance between these axle/axes is in all 0.041 h, within these limits can move the neutral axle/axis during change n from 1 to ∞. The error in the determination of neutral axle/axis, obtained with above method of its determination indicated, completely little affects the value of the bending moment at given rate of change in the curvature and on the value of the greatest voltage.

Subsequently we will recommend for solving the task of flexure the application/use of an exponential law of creep. If torque/moment is sufficiently great, then the voltages at each point of section/cut are approximately proportional σ_e, we must determine neutral axle/axis from the condition

$$\int_{F_1} \sigma_e dF = \int_{F_2} \sigma_e dF. \qquad (38.1)$$

Here F_1 and F_2 - parts of the section/cut, divided by neutral axle/axis. Virtually in this case, one should enter as follows. Cross-sectional area is divide/marked off into k of strips ΔF_i, which are labeled ΔF_1, ΔF_2, ..., for example, from below. Is calculated the

area of each strip, it is multiplied by appropriate average value σ_e. Further is constructed plotted function, which is $\int \sigma_e dF$, т. е. $\sum_{i=1}^{i} \sigma_{ei} \Delta F_i$, ordinates are set aside against the boundary/interfaces of the corresponding sections. Is measured the final ordinate, which corresponds i=k, it is divided in half, and is located that point on the graph of the integral function whose ordinate is equal to the half of final.

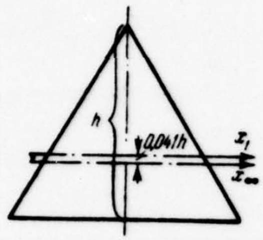

Fig. 74. Diagram of airfoil/profile in the form of isosceles triangle.

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This is determined the position of the neutral axle/axis Ox. Now does not comprise the work to calculate values β, $\varkappa_e$ and M_e in formula (36.5), which retains force. Only formulas (37.2) will have to replace with following:

$$M_e=\int_{-h_1}^{h_2}\sigma_e b|y|dy,\ \ln\beta=\frac{\int_{-h_1}^{h_2}\sigma_e b|y|\ln\frac{e_0}{e_e}\frac{|y|}{h}dy}{\int\sigma_e b|y|dy},\quad (38.2)$$

in this case as before

$$\varkappa_e=\frac{e_0}{\beta h}.$$

Since the laying out of section/cut to strips is already made, to us remains to note the centers of gravity of these strips, to calculate their distances from the obtained neutral axle/axis and to calculate by the numerical integration of value, that figure as in

relationship/ratios (38.2).

§39. Approximate method of the calculation of rods to flexure.

Under the power law of creep, the constant σ_n was determined from that condition that the creep rate when $\sigma = \sigma_n$ takes one and the same value independent of temperature. If the value of index n is sufficiently high, then the value of effective stress little the speed-sensing of creep. At constant temperature this leads to the fact that the stress distributions in the elongated and compressed zones are proved to be close to uniform ones. At variable over section/cut temperature the voltage at each point of section/cut is approximately proportional σ_n. Thus, $\sigma = \lambda\sigma_n$ in tension area, and $\sigma = -\lambda\sigma_n$ in compressed zone. The position of neutral axle/axis is found from the condition

$$\int_{F_1} \sigma_n \, dF - \int_{F_2} \sigma_n \, dF = 0. \tag{39.1}$$

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Here F_1 and F_2 - area of elongated (y>0) and of of compressed (y<0) of the parts of the section/cut respectively. Now the bending moment is calculated as follows:

$$M = \int_F \sigma y \, dF = \lambda M_*.$$

In this case,

$$M_* = \int_{F_1} \sigma_n y \, dF - \int_{F_2} \sigma_n y \, dF. \tag{39.2}$$

Value M_* is calculated by numerical integration. Now

$$\sigma = \pm \lambda\sigma_n = \pm M \frac{\sigma_n}{M_*}.$$

The velocity potential, arriving per the unit of the length of rod, is expressed as follows (§21):

$$\tilde{\Phi} = \int_F \Phi\left(M \frac{\sigma_n}{M_*}\right) dF.$$

Consequently, according to the theorem of Castiglianc

$$\varkappa = \frac{1}{M_*}\left[\int_{F_1} \sigma_n v\left(M \frac{\sigma_n}{M_*}\right) dF - \int_{F_2} \sigma_n v\left(M \frac{\sigma_n}{M_*}\right) dF\right]. \quad (39.3)$$

Let us use this formula to the case when temperature is constant and σ_n is a constant. From condition (39.1) it follows that the neutral axle/axis divides cross-sectional area into equivalent parts. Let us note the centers of gravity of these parts; let the distance between them there will be c (Fig. 75). Then on formula (39.2)

$$M_* = \frac{\sigma_n F c}{2}, \qquad F = F_1 + F_2, \qquad (39.4)$$

and on formula (39.3)

$$\varkappa = \frac{2}{c} v\left(\frac{2M}{cF}\right).$$

Under the power law of creep

$$\varkappa = \frac{2e_n}{c}\left(\frac{2M}{cF\sigma_n}\right)^n.$$

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The obtained formula can be recorded in the form (36.3), namely:

$$\varkappa=\varkappa_n\left(\frac{M}{M_n}\right)^n,$$

if is accepted

$$\varkappa_n=\frac{2\varepsilon_n}{c},\qquad M_n=\frac{cF\sigma_n}{2}. \tag{39.5}$$

However, if we want to compare approximate solution with precise, us one should to define $\varkappa_n$ then just as this was done into §36, namely accept $\varkappa_n=\varepsilon_n/h$, then

$$M_n=\frac{cF\sigma_n}{2}\left(\frac{c}{2h}\right)^{1/n}.$$

Thus, for instance, for a rectangle we will obtain

$$M_n=2^{-1/n}\sigma_n bh^2.$$

Already with n=3 factor $2^{-1/n}=0{,}795$, whereas in exact solution (§36) figures as factor 2n/2n+1=0,858. Thus, the error in value determination of the torque/moment, necessary for report/communication to assigned/prescribed rate of change in the curvature composes 7,3o/o. This error rapidly is decreased with increase of n and it varishes with $n\to\infty$. Completely another position will be, if we begin to compare the rates, which correspond to the assigned/prescribed torque/moment. The ratic of the velocities, found from precise and approximation formulas, does not approach unity with increase n and the value of error with sufficiently large n can be as great as desired.

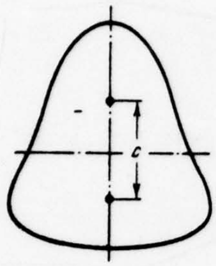

Fig. 75. To the approximate method of the calculation of rods to flexure.

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§40. Calculations for bending creep according to isochronal curves.

During calculations for creep of parts and elements of construction/designs, manufactured from such materials as copper and some austenitic steels when the department/separation of instantaneous plastic deformation from creep strain is hinder/hampered, it is necessary to use isochronal curves of creep (§16). Calculation according to isochronal curves is reduced to the fact that the curve of creep in coordinates $\sigma-\epsilon$ for each fixed value of t is accepted for the curve of plastic deformation, and the dependence between the bending moment and curvature is located exactly as is located instantaneous bending strain. The methods of

determining the instantaneous deformation were described into §§33 and 34; these methods literally are transferred to the task in question. As a result is located curvature k(t), that corresponds to the assigned/prescribed torque/moment of time t. Desiring to trace the process of creep in time, we must repeat calculation for the different values of t.

The volume of computations substantially is decreased, if is considered the similarity of isochronal curves and to considered valid formula (16.2). With t=0 formula (16.2) gives

$$\sigma = \varphi(e).$$

But this exists an equation of the curve of instantaneous deformation. Let us find the dependence of the bending moment on curvature with t=0:

$$M = M_0(k),$$

then for any moment of time, it will be

$$M = \frac{M_0(k)}{1 + at^{\beta}}. \quad (40.1)$$

Isochronal the curves of the dependences of torque/moment on curvature are proved to be similar. It is certain, formula (40.1) is used if and only if temperature over section/cut is constant.

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§41. Determination of the sagging/deflections of beams.

After is establish/installed the dependence between curvature and the bending moment, the determination of the sagging/deflection of beam is reduced to the integration of the equation

$$\frac{d^2 f}{dx^2} = k, \qquad (41.1)$$

or

$$\frac{d^2 \dot{f}}{dx^2} = \varkappa \qquad (41.2)$$

in the case of creep. Here x - coordinate, calculated in the direction of the axle/axis of beam, f - sagging/deflection, $\dot{f}$ - rate of sagging/deflection.

During calculation according to isochronal curves, it is necessary to determine the instantaneous sagging/deflection f_0, sagging/deflection at the any moment of the time

$$f = f_0(1 + at^{\beta}). \qquad (41.3)$$

The computation technique of instantaneous sagging/deflections and rates of sagging/deflection in the case of creep one and the same; therefore we will not examine these tasks separately. For certainty let us here speak about the rates of sagging/deflection in the case of steady-state creep.

For the definition of the rates of sagging/deflections in separate structural sections we can use the formula of Moore which is

derive/concluded completely in the same manner as in the theory of elastic beams, and has the following form:

$$f = \int \varkappa m_0 \, dx. \quad (41.4)$$

Here m_0 - torque/moment from the unit power, applied in that section/cut, in which searches for the sagging/deflection.

For computing integral (41.4) can be used the known graphoanalytical procedure, called the rule of the multiplication of diagram/curves (method of Vereshchagin). Is constructed the diagram/curve of value $\varkappa$ depending on x, diagram/curve is divide/marked off into the sections, which correspond to the linear sections of diagram/curve m_0, is determined the area of each section, is located its center of gravity. The area of section is multiplied by the ordinate of diagram/curve m_0, which corresponds to the abscissa of the center of gravity of this section. The sum of the obtained products is integral (41.4).

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Example of 41.1. The cantilever beam of rectangular cross section (b=10 mm, 2h=20 mm, l=250 mm) is bent by concentrated force of Q, applied at the free end of the arm. Permissible free-running speed of the sagging of the free end of the beam [f] =0,025 mm/s. Material of beam VT-14, temperature of 600°C. To determine the

permissible amount of force Q.

Rate of change of the curvature in section/cut, which is located at a distance of x from the loaded end cf the arm:

$$\varkappa(x) = \varkappa_n \left(\frac{Q}{M_n}\right)^n x^n,$$

$$\varkappa_n = \frac{\varepsilon_n}{h} = 10^{-5}\ \text{сек}^{-1}\text{мм}^{-1},\ ^{(1)}$$

$$M_n = \frac{2n}{2n+1}\sigma_n bh^2 = \frac{2\cdot 4{,}80}{2\cdot 4{,}80+1}\cdot 17{,}35\cdot 10\cdot 10^2 = = 1{,}57\cdot 10^4\ \text{кГ}\cdot\text{мм}.\ ^{(2)}$$

Key: (1). s^{-1} mm^{-1}. (2). kgf•mm.

Rate of the sagging of the free end

$$f = \int_0^l \varkappa(x) m_0(x)\,dx = \frac{\varkappa_n}{M_n^n} Q^n \int_0^l x^{n+1}\,dx = \frac{\varkappa_n Q^n}{(n+2) M_n^n} l^{n+2}.$$

Consequently,

$$Q = M_n\left[\frac{(n+2)[f]}{\varkappa_n}\right]^{1/n}\frac{1}{l^{\frac{n+2}{n}}} = = 1{,}57\cdot 10^4\left[\frac{(4{,}80+2)\,0{,}025}{10^{-5}}\right]^{\frac{1}{4{,}80}}\frac{1}{250^{\frac{4{,}80+2}{4{,}80}}} = 48{,}0\ \text{кГ}.\ ^{(1)}$$

Key: (1). kgf.

The bending moment in bearing edge will be M=48•250=12000 kgf•mm

and rate of change of the curvature in bearing edge able of the steady-state **creep**

$$\varkappa=\varkappa_n\left(\frac{M}{M_n}\right)^n=10^{-5}\left(\frac{12\cdot10^3}{15,7\cdot10^3}\right)^{4,80}=0,274\cdot10^{-5}\ \text{сек}^{-1}.$$

Key: (1). s^{-1}.

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Creep rate of **outermost fiber**

$$\varepsilon=\varkappa y=2,74\cdot10^{-5}\ \text{сек}^{-1}.$$

Key: (1). s 1.

Thus, those volumes of the bent rod, which make the greatest contribution to sagging/deflection, are proved to be loaded in the range where the constants of creep are determined.

Example of 41.2. Cantilever beam by length l=200 mm, that has round cross-section d=20 mm, is loaded by concentrated force of Q at free end. Temperature is constant over section/cut and varies linearly along the length. in framing T=600°, at the free end T=800°. To determine the permissible amount of force Q, if the permissible free-running speed of the sagging of free end [f] =0,003 mm/s. Material of beam OT-4.

For titanium alloy OT-4 in this temperature range, the exponent in the law of creep does not depend on temperature and is equal [illegible] 3. Therefore, the dependence of the velocity of the free end of the arm on the amount of force Q (see the preceding/previous example) is equal to

$$[\dot{f}]=\int_0^l \varkappa(x)\, m_0\, dx=\varkappa_n Q^n \int_0^l \frac{x^{n+1}}{M_n^n}\, dx=\varkappa_n Q^3 \int_0^l \frac{x^4}{M_n^3}\, dx,$$

$$\varkappa_n=\frac{\varepsilon_n}{R}=10^{-5},$$

$$M_n=\mu(n)\, R^3 \sigma_n.$$

Value $\mu(n)$ is found through table on page 134, $\mu(3)=1{,}091$.

The computation of integral $J=\int_0^l \frac{x^4}{M_n^3}\, dx$ is given in table, J=0,09476 kgf$^{-3}\cdot$mm^2. Consequently,

$$Q=\sqrt[3]{\frac{[\dot{f}]}{\varkappa_n J}}=\sqrt[3]{\frac{0{,}003}{10^5\cdot 0{,}09476}}=14{,}72\ \kappa\Gamma.$$

Key: (1). kgf.

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(1) x мм	T °C	(2) σ_n кГ/мм²	(3) $M_n \cdot 10^{-3}$ кГ·мм	(4) $M_n^3 \cdot 10^{-12}$ кГ³·мм³	(5) $x^4 \cdot 10^{-8}$ мм⁴	(6) $\frac{x^4}{M_n^3} 10^4$ кГ⁻³·мм
200	600	10,95	11,95	1,71	16,00	9,20
190	610	10,70	11,68	1,59	13,03	8,21
180	620	10,50	11,46	1,50	10,50	7,00
170	630	9,78	10,68	1,22	8,35	6,84
160	640	8,91	9,73	0,921	6,55	7,12
150	650	8,12	8,86	0,696	5,06	7,28
140	660	7,42	8,10	0,531	3,84	7,23
130	670	6,76	7,38	0,402	2,86	7,13
120	680	6,03	6,58	0,285	2,07	7,27
110	690	5,50	6,00	0,216	1,464	6,78
100	700	5,02	5,48	0,1646	1,000	6,08
90	710	4,57	4,98	0,1235	0,656	5,31
80	720	4,07	4,44	0,0899	0,410	4,56
70	730	3,72	4,07	0,0674	0,240	3,56
60	740	3,39	3,70	0,0506	0,1296	2,57
50	750	3,09	3,37	0,0383	0,0625	1,63
40	760	2,82	3,08	0,0292	0,0256	0,88
30	770	2,51	2,74	0,0206	0,0081	0,55
20	780	2,24	2,45	0,01471	0,0016	0,15
10	790	2,00	2,18	0,01036	0,0001	0,01
0	800	1,82	1,99	0,00788	0	0

(7) $J = 94{,}76 \cdot 10^{-3}$ кГ⁻³мм².

Key: (1). mm. (2). kg/mm². (3). kgf•mm. (4). kgf³•mm³. (5). mm⁴. (6). kgf ³•mm. (7). kgf ³mm².

Example of 41.3. Rod (Fig. 76) of circular cross section (R=6 mm, the relation of radii γ=0,8) is made from the alloy EP-202 and is loaded by the evenly distributed load. It is required to determine the permissible value of load q depending on the temperature of rod (in the range of 850-1150°C), if it is heated evenly and the permissible free-running speed of the displacement of the free end $\dot{f}$=0,01 mm/s, l=100 mm.

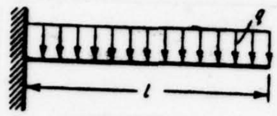

Fig. 76. Tc example of 41.3. Diagram of the loading of rod.

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Rate of the displacement of the free end cf the rod in the steady state

$$\dot{f}=\int_0^l \varkappa m_0\,dx=\frac{\varkappa_n}{M_n^n}\int_0^l M^n m_0\,dx=\frac{\varkappa_n q^n}{2^n M_n^n}\int_0^l x^{2n+1}\,dx=\frac{\varkappa_n q^n l^{2(n+1)}}{2^{n+1}(n+1)M_n^n},$$

and therefore

$$q=M_n\left[\frac{2(n+1)R}{\varepsilon_n l^2}\dot{f}\right]^{1/n}\frac{2}{l^2}. \qquad (*)$$

For cross section in the form of the ring

$$M_n=\mu(n)R^3\sigma_n(1-\gamma^3).$$

Value $\mu(n)$ one should take the curve/graph cf Fig. 73, after dividing ordinates curve on $(1-\gamma^3)$. The values of the necessary constants, value $\mu(n)$ and the obtained on formula (*) values q are brought to table.

T °C	n	(1) σ_n кГ/мм^2	μ	$M_n \cdot 10^{-3}$ кГ·мм (2)	(2) q кГ/мм	$M_{max} \cdot 10^{-3}$ кГ·мм (2a)	$\varepsilon_n \left(\frac{M_{max}}{M_n}\right)^n \cdot 10^4$ (3) сек$^{-1}$
850	8,98	31,3	1,243	4,12	1,405	7,02	$1,22 \cdot 10^2$
900	8,30	21,9	1,237	2,87	0,582	2,91	1,15
950	8,05	11,9	1,234	1,56	0,313	1,56	1,02
1000	4,90	5,96	1,176	0,742	0,140	0,698	0,740
1050	4,70	3,72	1,173	0,462	0,0607	0,303	0,135
1100	4,62	2,71	1,167	0,335	0,0450	0,225	0,159
1150	4,54	2,14	1,165	0,265	0,0371	0,186	0,197

Key: (1). kg/mm². (2). kgf/mm. (2a). kgf•mm. (3). s^{-1}.

In the last/latter two graph/counts of table, are contained values M_{max} and $\varepsilon_n (M_{max}/M_n)^n$. The latter is the creep rate of the peripheral filaments of rod in the maximally loaded section/cut which makes the greatest contribution to sagging/deflection.

Example of 41.4. The rod of round cross section (r=10 mm), manufactured from copper M-1, is loaded, as shown in Fig. 77a. To find the dependence of bending deflection from time (to 1000 s). Rod is heated evenly to temperature of 600°, q=0,25 kgf/mm.

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Let us find first instantaneous sagging/deflection in the center of rod. Dependence of torque/moment on curvature (§33):

$$M = r^3\psi(k_0r).$$

Utilizing results of example of 33.3, we will obtain

$$\psi(k_0 r) = 1{,}015\,\varphi(k_0 r).$$

After using exponential approximation $\varphi(\xi) = K\xi^m$, we will obtain

$$M = 1{,}015 K r^{m+3} k_0^m,$$

and

$$k_0 = \mu M^{1/m}, \qquad (*)$$

where

$$\mu = \left[\frac{1}{1{,}015 K r^{m+3}}\right]^{1/m} = \left[\frac{1}{1{,}015 \cdot 22{,}8 \cdot 10^{0.457+3}}\right]^{1/0{,}457} = 8{,}85 \cdot 10^{-11}.$$

Values K and m are undertaken on the curve/graph of Fig. 130. Instantaneous sagging/deflection in section/cut x=1/2 is determined

$$f_0 = \int_0^l k_0(x)\, m_0(x)\, dx,$$

where m_0 - bending moment from the unit power, applied in section/cut x=1/2. This integral is conveniently taken with the aid of the rule of the multiplication of the diagram/curves:

$$\int_0^l k_0(x)\, m_0(x)\, dx = \Omega_k m_{0\text{ц.т.}}.$$

Here Ω_k - the area of diagram/curve $k_0(x)$, $m_{0\text{ц.т.}}$ - the ordinate of diagram/curve $m_0(x)$ (Fig. 77b) under the center of gravity of diagram/curve $k_0(x)$.

Area Ω_k it is easy to calculate, if to note that the dependence of the bending moment on x (Fig. 77c) is expressed by the square parabola

$$M = \frac{ql}{2}\left(x - \frac{x^2}{l}\right).$$

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Then as a result of (*) equation curved, that limits the diagram/curve of curvature (Fig. 77c) in coordinates η, ξ, shown on figure, exists an equation of the parabola of degree 2/m:

$$k'(\xi) = \alpha |\xi|^{2/m}.$$

It is obvious that

$$k'\left(-\frac{l}{2}\right) = k'\left(\frac{l}{2}\right) = \alpha\left(\frac{l}{2}\right)^{2/m} = \mu\left[M\left(\frac{l}{2}\right)\right]^{1/m} = \mu\left(\frac{ql^2}{8}\right)^{1/m} = 8{,}85 \cdot 10^{-11}\left(\frac{0{,}25 \cdot 250^2}{8}\right)^{1/0{,}457} = 1{,}40 \cdot 10^{-3}.$$

Therefore one should accept

$$\alpha = \frac{1{,}40 \cdot 10^{-3}}{\left(\frac{250}{2}\right)^{2/0{,}457}} = 9{,}47 \cdot 10^{-13}.$$

Then the area, shaded in Fig. 77c by horizontal lines, will be

$$\Omega_k' = 2\alpha\int_0^{l/2} \xi^{2/m}\, d\xi = \frac{2\alpha}{\frac{2}{m}+1}\left(\frac{l}{2}\right)^{\frac{2}{m}+1} = 6{,}57 \cdot 10^{-2}$$

and

$$\Omega_k = lk'\left(\frac{l}{2}\right) - \Omega_k' = 250 \cdot 1{,}40 \cdot 10^{-3} - 6{,}57 \cdot 10^{-2} = 0{,}284.$$

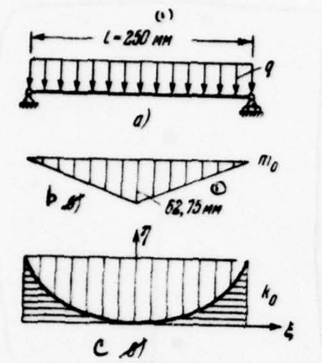

Fig. 77.

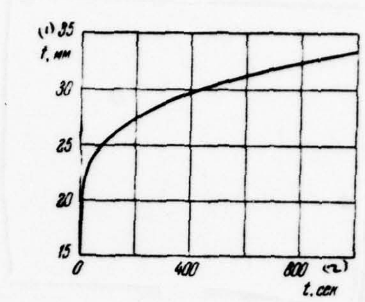

Fig. 78.

Fig. 77. To example of 41.4. Diagram of the loading of rod, diagram/curves m_0 and k_0.

Key: (1). mm.

Fig. 78. To example of 41.4. Change in the sagging/deflection with time.

Key: (1). mm. (2). s.

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The center of gravity of diagram/curve $k_0(x)$ is located with $x=l/2$, and since

$$m_0\left(\frac{l}{2}\right)=\frac{l}{4}=62{,}75 \text{ mm}.$$

that

$$f_0 = \Omega_k \frac{l}{4} = \\ = 0{,}284 \cdot 62{,}75 = 17{,}8 \text{ mm.}$$

The dependence of sagging/deflecticn on time is located through formula (41.3)

$$f = f_0(1 + at^{0.3}),$$

in which should be assumed a=0,11 (see Fig. 131).

The curve f(t) is constructed in Fig. 78.

Example of 41.5. Rod (Fig. 79a), material and size/dimensions of which the same as in example cf 41.4, is heated unevenly along the length, the temperature of its left end of 300°, rightist 800°C (Fig. 79b) q=0,1 kgf/mm. Is required to determine the maximum sagging/deflection of rod with t_0=0 and t'=100 s.

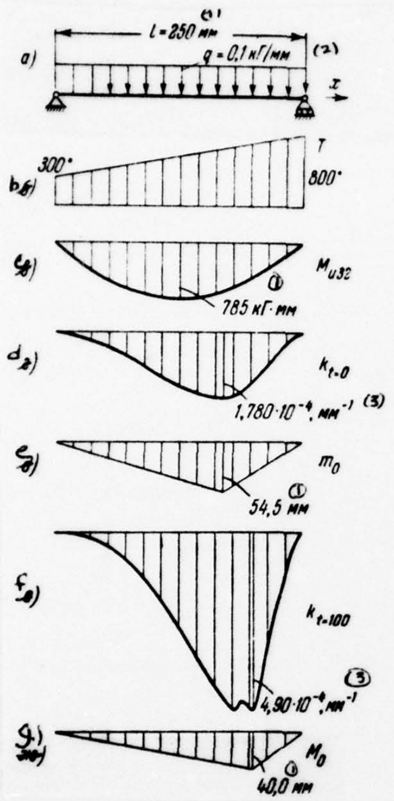

Fig. 79. To example of 41.5. Diagram of loading, diagram/curves T, $M_{изг}$, k_{t-0}, m_0, k_{t-100}, M_0.

Key: (1). mm. (2). kgf/mm. (3). mm⁻¹.

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Since the characteristics of the material of rod vary with temperature along the length of rod, then the dependence of curvature on the bending moment will be also changed along the length of rod,

and instead of formula (*) cf example of 41.4 one should record

$$k_0=\left[\frac{1}{\chi(x)\,K(x)\,r^{m(x)+3}}\right]^{\frac{1}{m(x)}} M^{\frac{1}{m(x)}}=\mu(x)\,M^{\frac{1}{m(x)}}. \qquad (**)$$

Value $\chi(x)$ **is changed weakly (see example of 33.3), it can be** accepted by constant and equal to 1.02.

Dependence of **torque/moment on length (Fig. 79c):**

$$M=\frac{ql}{2}\left(x-\frac{x^2}{l}\right).$$

In the following table are **given values K(x) and m(x),** obtained reference data for temperatures in accordance with diagram/curve Fig. 79b. Further in the same table are given the calculated values μ and k_0. Diagram/curve $k_0(x)$ it is constructed in Fig. 79d. Maximum sagging/deflection is obtained, as is evident, with x=170 mm. The diagram/curve of the bending moment from the unit power, applied in the section/cut of maximum sagging, has the form of Fig. 79e. The value of bending deflection will be

$$f_0=\int_0^l k_0(x)\,m_0(x)\,dx.$$

This integral is calculated **in table. Is obtained** f_0=0,854 by mm. To use formula (41.3) for determining the sagging/deflection f(t') in this case is impossible, since value a in it is not constant, it depends on x. **Therefore, in the ninth column** of the table, is determined curvature $k_{t=100}$ **(x) according to the formula**

$$k(x)=k_0(x)\left[1+a(x)\cdot(100)^{\beta}\right].$$

The obtained diagram/curve of curvature is constructed in Fig. 79f. The section/cut of maximum sagging during creep is misaligned in the direction of the more heated end of the rod and is located with t=100 at point x=200 mm. Further computations are analogous carried out above, they are contained in the same table and give $f_{t=t'} =$ 1,515 mm.

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(1) x мм	(2) K кГ/мм²	m	$\mu \cdot 10^{11}$	(5) M кГ·мм	(4) $k_0 \cdot 10^4$ мм$^{-1}$	(1) m_0 мм	$k_0 M_0 \cdot 10^4$	(4) $k(t')\,10^4$ мм$^{-1}$	(1) m_0 мм	$km_0 \cdot 10^4$
0	42,7	0,497	9,33	0	0	0	0	0	0	0
10	42,0	0,495	9,25	120	0,015	3,20	0,048	0,015	2,0	0,030
20	41,3	0,493	9,20	230	0,056	6,41	0,358	0,056	4,0	0,224
30	40,6	0,490	9,10	330	0,126	19,62	1,222	0,126	6,0	0,775
40	39,9	0,188	9,06	420	0,214	12,8	2,74	0,214	8,0	1,71
50	39,2	0,485	8,98	500	0,331	16,0	5,30	0,372	10	3,72
60	38,2	0,483	8,93	570	0,457	19,2	8,78	0,538	12,0	6,45
70	36,9	0,480	8,88	630	0,603	22,4	13,50	0,742	14,0	10,37
80	35,5	0,477	8,83	680	0,776	25,6	19,85	1,022	16,0	16,35
90	34,1	0,475	8,80	720	0,912	28,8	26,80	1,258	18,0	22,7
100	32,4	0,472	8,77	750	1,108	32,0	35,5	1,622	20,0	32,5
110	30,8	0,469	8,75	770	1,260	35,3	44,5	2,000	22,0	44,0
120	28,6	0,466	8,75	780	1,412	38,5	54,4	2,46	24,0	59,0
130	26,7	0,464	8,80	780	1,513	41,7	63,1	2,82	26,0	73,3
140	24,7	0,461	8,83	770	1,621	44,9	72,9	3,24	28,0	90,7
150	22,8	0,457	8,85	750	1,738	48,1	83,5	3,80	30,0	114,0
160	21,6	0,453	8,77	720	1,738	51,3	88,2	4,17	32,0	133,2
170	20,5	0,449	8,67	680	1,780	54,5	97,0	4,57	34,0	155,0
180	19,3	0,445	8,61	630	1,738	47,7	82,9	4,90	36,0	176,2
190	18,2	0,441	8,53	570	1,513	40,8	61,8	4,57	38,0	173,4
200	17,1	0,436	8,44	500	1,320	34,0	44,9	4,90	40,0	196,0
210	16,0	0,431	8,35	420	1,000	27,3	27,9	3,55	32,0	113,5
220	14,9	0,425	8,22	330	0,692	20,4	14,1	2,63	24,0	63,2
230	13,8	0,418	8,07	230	0,363	13,6	4,94	1,55	16,0	24,8
240	12,8	0,411	7,85	120	0,093	6,80	0,630	0,44	8,0	3,52
250	11,8	0,403	7,66	0	0	0	0	0	0	0
							$\sum = 854{,}37$			1514,65

Key: (1). mm. (2). kg/mm². (3). kgf/mm. (4). mm⁻¹.

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§42. Creep statically indeterminable beams.

The calculation statically indeterminable systems to creep it is more convenient anything to perform with the aid of the Castigliano theorem. Formula for the potential of bending creep will be following:

$$\tilde{\Phi} = \int \Phi(M)\,dx,$$
$$\frac{\partial \Phi}{\partial M} = \varkappa. \qquad (42.1)$$

The value of the bending moment depends on external loads and on the unknown reactions X_1, X_2, ... Equalizing to zero derivatives the potential of creep on excess unknowns, we obtain system of equations for their determination

$$\int \varkappa(M)\frac{\partial M}{\partial X_1}\,dx = 0,$$
$$\int \varkappa(M)\frac{\partial M}{\partial X_2}\,dx = 0.$$

Actually, these equations are proved to be complex, and can be solved them only in the simplest cases, for example under the power law of creep and at whole (too great) values of index n. The approximate procedure lies in the fact that the distribution of the bending moments is received the same as in the limiting condition of the perfectly plastic beam, whose limit moment is equal to M_n. Let us examine the perfectly plastic beam, for which the limit moment is equal to λM_n. Generally speaking, value M_n - is alternating/variable along the length of beam. If the degree of static indeterminancy/uncertainty is equal p, then limiting condition sets in during appearance p+1 of a hinge joint of yield.

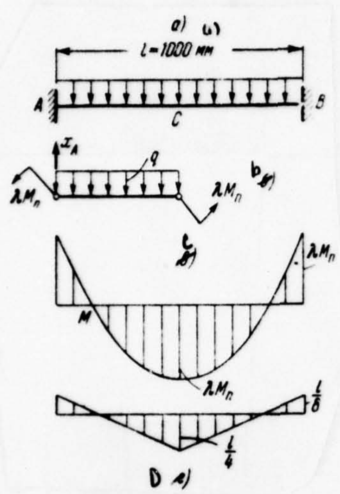

Fig. 80. To example of 42.1. Diagram of the loading of beam as a whole and of [redacted] it left half; diagram/curves M and m_0. (?)

Key: (1). mm.

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Equalizing the work of torque/moments in plastic hinge joints to the work of external forces, we will obtain the equation, from which is determined λ. The diagram of the layout of plastic hinge joints can be selected not by only form, one should accept that diagram which corresponds to the greatest value λ. After this it is constructed the diagram/curve of the bending moments in the usual way and sagging/deflections can be defined in the manner that this is stated

in §41.

The special feature/peculiarity solving of the tasks of creep in comparison with the common tasks of limit equilibrium lies in the fact that due to variability M_n in the case of variable temperature the plastic hinge joints can appear in different section/cuts and their position previously not is obvious.

Example of 42.1. The beam (Fig. 80a), which has airfoil/profile PR-106 No. 12 (Fig. 81) and manufactured from CT-4, is loaded by evenly distributed load q=0,08 kgf/mm. The temperature distribution over section/cut is display in Fig. 81. It is required to determine the free-running speed of sagging/deflection in the center of flight/span.

Let us find first the distribution of the bending moment along the length of the beam, the degree of static indeterminancy/uncertainty of which is equal to 2. Let us count the distribution of torque/moments as such, what it is accepted for a perfectly plastic beam in limiting condition.

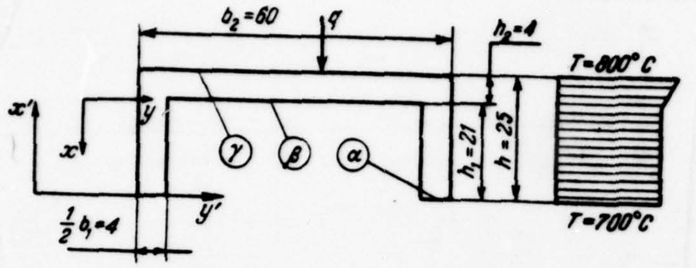

Fig. 81. To example of 42.1. Airfoil/profile of beam and the temperature distribution.

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Plastic hinge joints in the beam in question appear at first in section/cuts A and B, beam becomes statically determinable. The following section/cut in which torque/moment it reaches limiting value and beam it transfer/converts into limiting condition, will be section/cut in the center of flight/span. Let us examine the equilibrium of the left half beam under the action of bending moments λM_n and of the external loads, indicated in Fig. 80b:

$$\sum M_C = -2\lambda M_n + X_A \frac{l}{2} - q\frac{l^2}{8} = 0,$$

$$X_A = \frac{ql}{4} + \frac{4\lambda M_n}{l},$$

$$\sum Q = X_A - \frac{ql}{2} = -\frac{ql}{4} + \frac{4\lambda M_n}{l} = 0,$$

$$\lambda = \frac{ql^2}{16 M_n}.$$

Moment diagram it is depicted in Fig. 80c. Equation M(x) we will obtain, by integrating the equation

$$\frac{d^2M}{dx^2} = q$$

under the specified above boundary conditions. It takes the form

$$M(x) = \frac{ql^2}{16} - \frac{ql}{2}x + \frac{qx^2}{2}.$$

The dependence of curvature on torque/moment let us search for the approximation method, presented into §39. Let us preliminarily note that the computation of integral (39.3)

$$\varkappa = \frac{1}{M_*}\left[\int\limits_{F_1} \sigma_n v\left(\frac{M}{M_*}\sigma_n\right)dF - \int\limits_{F_2} \sigma_n v\left(\frac{M}{M_*}\sigma_n\right)dF\right]$$

by numerical integration is bulky operation, since the determination of dependence $\varkappa(M)$ requires the multiple repetition of numerical integration at different values of M. At the same time it is easy to see (see Fig. 102) that the value of index n in the range of temperatures of 700-800°C is constant and equal to 3, but dependence $\sigma_n(T)$ for the same temperature range, constructed in Fig. 82 according to of these Fig. 102, it is close to linear.

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The linear dependence

$$\sigma_n = \sigma_{n0} - sT$$

is carried out in Fig. 82 in such a way that

$\sigma_{n0} = 26{,}0$ **kg/mm²** $s = 0{,}0307$ **kgf/mm²•deg.**

For determining the position cf neutral axle/axis, let us calculate first the integral

$$I = I_1 + I_2,$$

$$I_1 = \int\limits_{F_1} \sigma_n\, dF = F_1\sigma_n(\alpha) = b_1h_1\sigma_n(\alpha),$$

$$I_2 = \int\limits_{F_2} \sigma_n\, dF = b_2 \int\limits_{h_1}^{h} \{\sigma_{n0} - s\,[T(\beta) + \vartheta(x' - h_1)]\}\, dx' =$$

$$= b_2 \int\limits_{h}^{h} [\sigma_n(\beta) - s\vartheta(x' - h_1)]\, dx' =$$

$$= b_2h_2\sigma_n(\beta) - b_2s\vartheta\frac{h_2^2}{2} = b_2h_2\sigma_n(\beta)\left[1 - \frac{1}{2}\frac{s\vartheta h_2}{\sigma_n(\alpha)}\right].$$

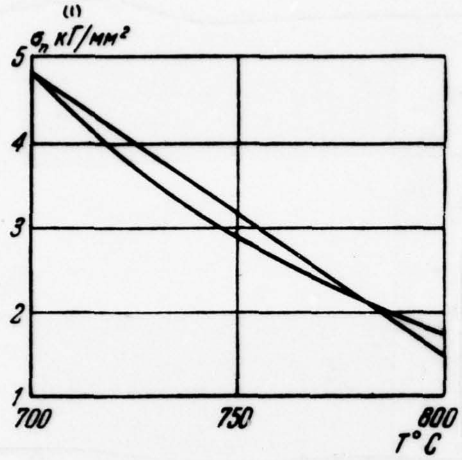

Fig. 82. To example of 42.1. Curves $\sigma_n(T)$ and its linear approximation for alloy OT-4.

Key: (1). kg/mm².

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Here $\vartheta = T(\gamma) - T(\beta)/h_2$, and α, β, γ are related the corresponding values to planes α, β, γ in Fig. 81.

We have

$$\vartheta = 25 \text{ град/мм}^{(1)}, \quad I_1 = 21 \cdot 8 \cdot 4{,}89 = 822 \text{ кГ}^{(2)},$$
$$I_2 = 60 \cdot 4 \cdot 4{,}89 \left[1 - \frac{1}{2}\,\frac{0{,}0307 \cdot 25 \cdot 4}{4{,}89}\right] = 802 \text{ кГ}^{(2)}.$$

Key: (1). deg/mm. (2). kgf.

Neutral axle/axis, strictly speaking, divides section/cut so that integral I is proved to be broken to two equal parts. However, in our case $I_1 \simeq I_2$, and it is possible to consider neutral y axis, which lies at plane β. Computations in that case are simplified, and additional error will be small. Then

$$M_{*} = \int_{F_1} \sigma_n x\, dF - \int_{F_*} \sigma_n\, dF =$$
$$= b_1 \int_0^{h_1} \sigma_n(\beta)\, x\, dx + b_2 \int_0^{h_2} [\sigma_n(\beta) - s\vartheta x]\, x\, dx =$$
$$= \sigma_n(\beta) \left\{ \frac{b_1 h_1^2}{2} + \frac{b_2 h_2^2}{2} \left[1 - \frac{2}{3}\,\frac{s\vartheta h_1}{\sigma_n(\beta)}\right]\right\} =$$
$$= 4{,}89 \left\{ \frac{8 \cdot 21^2}{2} + \frac{60 \cdot 4^2}{2} \left[1 - \frac{2}{3}\,\frac{3{,}07}{4{,}89}\right]\right\} = 1{,}00 \cdot 10^4 \text{ kgf mm}.$$

We find now the dependence of the velccity of curvature on the bending moment:

$$\varkappa = \frac{1}{M_*}\left[\int_{F_1} \sigma_n v\left(\frac{M}{M_*}\sigma_n\right)dF - \int_{F_2} \sigma_1 v\left(\frac{M}{M_*}\sigma_n\right)dF\right] =$$
$$= \frac{e_n}{M_*}\left[b_1h_1\sigma_n(\beta)\left(\frac{M}{M_*}\right)^n + \right.$$
$$\left. + b_2h_2\sigma_n(\beta)\left(\frac{M}{M_*}\right)^n - \frac{1}{2}b_2h_2^2 s\vartheta\left(\frac{M}{M_*}\right)^n\right] =$$
$$= \frac{e_n}{M_*}\left(\frac{M}{M_*}\right)^n \sigma_n(\beta)\left\{b_1h_1 + b_2h_2\left[1 - \frac{1}{2}\frac{s\vartheta h_2}{\sigma_n(\beta)}\right]\right\} =$$
$$aM^3 \text{ mm}^{-1} \text{ s}^{-1}.$$

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Here

$$a = 0{,}1624 \text{ kg}^{-3} \text{ mm}^{-4} \text{ s}^{-1}.$$

The rate of sagging/deflection in the center of flight/span is determined by the integral of Moore

$$\dot{f} = \int_0^l \varkappa(x)\, m_0(x)\, dx,$$

which by symmetry strength can be recorded in the form

$$\dot{f} = 2\int_0^{l/2} \varkappa(x)\, m_0(x)\, dx$$

cr, if is introduced $\xi = x/l$,

$$f = 2l \int_0^{1/2} \varkappa(\xi)\, m_0(\xi)\, d\xi.$$

The dependence of torque/moment on length is found earlier:

$$M(\xi) = \frac{ql^2}{16}(1 - 8\xi + 8\xi^2),$$

consequently

$$\varkappa(\xi) = \frac{q^3 l^6 a}{2^{12}}(1 - 8\xi + 8\xi^2)^3.$$

The equation of moment diagram from the unit power, applied in the center of flight/span, will be

$$m_0 = \frac{l}{8}(1 - 6\xi)$$ kgf mm with $0 \leqslant \xi \leqslant \frac{1}{2}$.

Then

$$f = \frac{aq^3 l^8}{2^{14}} \int_0^{1/2} \Phi(\xi)\, d\xi,$$

where

$$\Phi(\xi) = (1 - 8\xi + 8\xi^2)^3 (1 - 6\xi) = \\ = 1 - 30\xi + 360\xi^2 - 2192\xi^3 + 7104\xi^4 - 11904\xi^5 + \\ + 9728\xi^6 - 3072\xi^7,$$

and integral value can be calculated:

$$\int_0^{1/2} \Phi(\xi)\, d\xi = 11{,}1.$$

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Thus, the rate of sagging/deflection will be

$$\dot{f} = \frac{aq^3l^8}{2^{14}} \cdot 11{,}1 = \frac{0{,}1624 \cdot 10^{-16} \cdot 0{,}08^3 \cdot 1000^8 \cdot 11{,}1}{2^{14}} = 5{,}62 \text{ mm/s.}$$

§43. Bending failure.

Approximate solution of the task of the determination of time to the failure of rod with flexure can be obtained as follows. Let us accept for certainty the section/cut of rod as rectangular with size/dimensions 2h and b, let us accept for x and y axes of the axis of the symmetry of section/cut will consider the bending moment M in plane yOz as constant and directed so that at the initial moment $\sigma>0$ with y>0. Let us assume

$$\xi = \frac{y}{h}.$$

With t=0 x axis is neutral axle/axis, in the range of positive ones ξ occurs the crack formation.

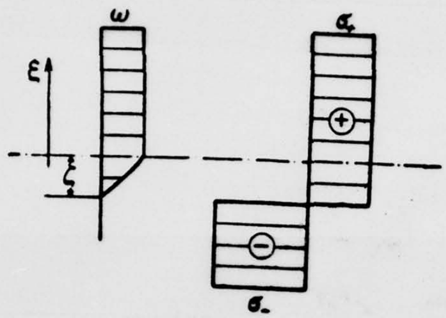

Fig. 83. To bending failure. Diagram/curves of the parameter of cracking and voltages, $\omega<1$; $\zeta<C65$.

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Consequently, the strength of materials of creep in tension area is decreased, and neutral axle/axis moves in the direction of negative ones ξ, so that at the moment of times t the coordinate of neutral axle/axis exists ζ (Fig. 83). Applying to this task the same approximate procedure of the estimation of time to failure which was used into §28 to frame, we must assume:

in the tension area

$$\sigma_+ = (1-\omega)\sigma, \quad -\zeta < \xi < 1,$$

in the compressed zone

$$\sigma_- = -\sigma, \quad -1 < \xi < \zeta.$$

Here $\sigma=\sigma(t)$ - the varying stress, determined from the condition of equilibrium, ω - parameter of damage.

Let us accept the power law of the accumulation of the damage:

$$\frac{\partial\omega}{\partial t}=\frac{\varepsilon_n}{e_*}\left(\frac{\sigma}{\sigma_n}\right)^n.$$

Let us for brevity designate through $\dot{\omega}$ the derivative on the changed time

$$\tau=\frac{\varepsilon_n}{e_*\sigma_n^n}t,$$

so that

$$\dot{\omega}=\sigma^n. \qquad (43.1)$$

Let us compose the condition of equality to zero longitudinal force in the section/cut:

$$\int_{-1}^{-\zeta}\sigma_-\,d\xi+\int_{-\zeta}^{1}\sigma_+\,d\xi=0$$

or

$$\int_{-\zeta}^{1}(1-\omega)\,d\xi+1+\zeta=0. \qquad (43.2)$$

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Neutral axle/axis drives off at each torque/moment the damaged zone from sound; therefore with $\xi=-\zeta$ must be $\omega=0$. Let us differentiate (43.2) on τ, taking into account this condition. Then under integral render/shows value $\dot{\omega}$, which as a result of (43.1) is function of one

only σ. Consequently, integration can be it will be fulfilled, and we will obtain

$$\zeta = \frac{1}{2}(1+\zeta)\dot{\omega}. \qquad (43.3)$$

Dividing variables and integrating taking into account initial condition $\zeta(0)=0$ and $\omega(0)=0$, let us find

$$\omega = 2\ln(1+\zeta) + \psi(\xi). \qquad (43.4)$$

Equation (43.1) indicates that $\dot{\omega}$ is function of one only τ. In the range of positive ones ξ where from the very beginning was elongation, the process of crack formation begins from torque/moment $\tau=0$, therefore, the integral of this equation depends only on τ. Thus,

$$\psi = 0 \ \text{при}^{(1)} \ 0 < \xi < 1.$$

Key: (1). with

In the range of negative ones ξ the crack formation at the particular point begins from that torque/moment when neutral axle/axis it passes through this point, and initial condition for integrating equation (43.1) will contain coordinate ξ. Therefore function $\psi(\xi)$ in (43.4) is different from zero. The value of this function will be located from that condition that with $\xi=-\zeta$ $\omega=0$. Substituting in (43.4), we will obtain:

$$\psi = -2\ln(1-\xi)^{(1)} \ \text{при} \ -\zeta < \xi < 0.$$

Key: (1) with

Thus:

$$\left.\begin{aligned}\omega &= 2\ln(1+\zeta), & 0<\xi<1,\\ \omega &= 2\ln\frac{1+\zeta}{1-\xi}, & -\zeta<\xi<1.\end{aligned}\right\} \qquad (43.5)$$

Value ω reaches limiting value $\omega=1$ simultaneously in the entire upper half section/cut. At this moment neutral axle/axis was misaligned relative to the axis of symmetry to value

$$\zeta_1 = e^{1/2} - 1 \approx 0{,}65.$$

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With this crack it is spread immediately to the half section/cut, the appropriate time t_1 can be accepted as the estimation of time to failure. For the determination of value t_1, let us compose the second equation of the equilibrium:

$$h^2 b\left[\int_{-1}^{-\zeta} \sigma_- \xi\, d\xi + \int_{-\zeta}^{1} \sigma_+ \xi\, d\xi\right] = M.$$

Let us assume

$$M = h^2 b \sigma_0;$$

here σ_0 - the voltage which corresponds to the bending moment M in rod from perfectly plastic material. Let us enter with this equation just as with the first equation of equilibrium; let us substitute expressions σ_- and σ_+, will differentiate with respect to τ and will integrate, after which let us eliminate $\dot{\omega}$ with the aid of (43.3).

We will obtain finally

$$\zeta(1+\zeta) = -\frac{\sigma_0}{\sigma^3}. \qquad (43.6)$$

Let us integrate this equation when with $\tau=0$, $\zeta=0$ and $\sigma=\sigma_0$. Let us find

$$\frac{\sigma_0}{\sigma}=\frac{1}{2}[3-(1-\zeta)^2]. \quad (43.7)$$

Now we can determine time to failure, integrating equation (43.1), in which $\dot{\omega}$ is replaced through ζ with the aid of (43.7). We will obtain

$$\tau_1=\frac{1}{\sigma_0^n 2^{n-1}}\int_0^{\zeta_1}\frac{[3-(1-\zeta)^2]^n}{1+\zeta}d\zeta. \quad (43.8)$$

With n=3 this formula gives

$$\tau_1=\frac{1}{2,72\sigma_0^3}.$$

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If we consider great voltage constant and to equal ones to σ_0, value τ it is equal to $1/4\sigma_0^3$ (see §8), the redistribution of voltages leads to an increase in the service life or the decrease of actual stress in relation to 1.135.

Strictly speaking, after the achievement of time τ_1, the rod still retains bearing capacity, and for determining the apparent time of complete destruction it was necessary to examine further process of crack propagation into the range of negative values ξ up to value $\xi=-1$. It is possible to show that this process continues very rapidly and true time to failure only a little exceeds value τ_1.

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Chapter V.

THICK-WALLED DUCTS.

§44. Closed thick-walled duct. Elasto-plastic state and calculation according to isochronal curves.

Thick-walled duct with an inside radius of a and with an outside radius of b is equipped by the bottoms and is subjected to the action of the internal pressure q. It is required to determine the instantaneous distribution of voltages and displacements. Material is assumed to be incompressible.

In the cylindrical coordinate system of voltage, they are designated $\sigma_r, \sigma_\theta, \sigma_z$, radial displacement is u, deformation respectively e_r and e_θ, deformation in axial direction $e_z=0$. Last/latter condition is satisfied for the incompressible material, which is establish/installed by the subsequent testing of solution. Formulas (18.6) are record/written as follows:

$$\left.\begin{aligned} e_r &= \frac{e(\sigma_o)}{\sigma_o}\left[\sigma_r - \frac{1}{2}(\sigma_\theta + \sigma_z)\right], \\ e_\theta &= \frac{e(\sigma_o)}{\sigma_o}\left[\sigma_\theta - \frac{1}{2}(\sigma_z + \sigma_r)\right], \\ e_z &= \frac{e(\sigma_o)}{\sigma_o}\left[\sigma_z - \frac{1}{2}(\sigma_r + \sigma_\theta)\right]. \end{aligned}\right\} \quad (44.1)$$

Here $e(\sigma_0)$ - the function, the graph by which is instantaneous deformation depending on the voltage in the case of plasticity or one of the isochronal curves of creep. From last/latter equation it follows:

$$\sigma_z = \frac{1}{2}(\sigma_r + \sigma_\theta). \qquad (44.2)$$

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Let us introduce expression (44.2) into formula (18.2) for S. We will obtain

$$\sigma_0 = \frac{\sqrt{3}}{2}(\sigma_\theta - \sigma_r). \qquad (44.3)$$

It is assumed that $\sigma_\theta > \sigma_r$; this assumption will be justified on course of solution. Deformations in cylindrical coordinates are expressed as follows:

$$e_r = \frac{du}{dr}, \qquad e_\theta = \frac{u}{r}.$$

From the condition of incompressibility, it follows:

$$e_r + e_\theta = \frac{du}{dr} + \frac{u}{r} = 0.$$

Integrating this equation, we will obtain

$$u = \frac{\sqrt{3}}{2}\frac{C}{r}.$$

Here integration constant is designated $\frac{\sqrt{3}}{2}C$. Hence

$$e_\theta = -e_r = \frac{\sqrt{3}}{2}\frac{C}{r^2}.$$

Let us deduct of seccnd equation (44.1) the first. We will cbtain

$$e_\theta - e_r = \frac{e(\sigma_o)}{\sigma_o}\frac{3}{2}(\sigma_\theta - \sigma_r)$$

cr

$$e(\sigma_o) = \frac{1}{\sqrt{3}}(e_\theta - e_r).$$

Hence, taking **into account (44.3),**

$$\sigma_\theta - \sigma_r = \frac{2}{\sqrt{3}}\varphi\left(\frac{e_\theta - e_r}{\sqrt{3}}\right)$$

cr

$$\sigma_\theta - \sigma_r = \frac{2}{\sqrt{3}}\varphi\left(\frac{C}{r^2}\right).$$

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The differential equation of equilibrium in the case of axial symmetry is **record/written as fcllcws:**

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \qquad (44.4)$$

Let us introduce the here oltained expressicn for $\sigma_\theta - \sigma_r$. **We will** cbtain

$$\frac{d\sigma_r}{dr} = \frac{2}{\sqrt{3}}\varphi\left(\frac{C}{r^2}\right)\frac{1}{r}.$$

Let us integrate this equation under initial **condition** $\sigma_r(a) = -q$. **Let** us find

$$\sigma_r = \frac{2}{\sqrt{3}}\int_a^r \varphi\left(\frac{C}{r^2}\right)\frac{dr}{r} - q.$$

Now to us it is convenient to replace the variable of integration. Let us accept for the independent variable value

$$x = \frac{C}{r^2}.$$

Then dr/r=-(1/2)(dx/x) and the formula written above takes the following form:

$$\sigma_r = -q - \frac{1}{\sqrt{3}} \int_{x_a}^{x} \frac{\varphi(x)\,dx}{x}. \qquad (44.5)$$

Utilizing second boundary condition $\sigma_r(b) = 0$, we will obtain the following equation:

$$q + \frac{1}{\sqrt{3}} \int_{x_a}^{x_b} \frac{\varphi(x)\,dx}{x} = 0. \qquad (44.6)$$

As a result of the determination of parameter x relation $\frac{x_b}{x_a} = \frac{a^2}{b^2}$, from equation (44.6) it is necessary to find unknown value x_a. After x_a is found, voltage σ_θ is calculated from the formula

$$\sigma_\theta = \frac{2}{\sqrt{3}} \varphi(x) + \sigma_r. \qquad (44.7)$$

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Formulas for determining the displacement will be: on the inside radius

$$u(a) = a \frac{\sqrt{3}}{2} x_a, \qquad (44.8)$$

on the outside radius

$$u(b) = \frac{a^2}{b} \frac{\sqrt{3}}{2} x_a. \qquad (44.9)$$

If function $\phi(x)$ is assign/prescribed graphically, then for the solution of equation (44.6) it is possible to recommend simple

graphic procedure. Let us assume

$$\frac{1}{\sqrt{3}}\int_0^x \frac{\varphi(x)\,dx}{x} = \Phi(x). \qquad (44.10)$$

Equation (44.6) is copied as follows:

$$\Phi(x_a) - \Phi\left(\frac{a^2}{b^2}x_a\right) = q. \qquad (44.11)$$

Plotted function $\Phi(x)$ it is easy to construct by numerical or graphical integration, after which for the assigned/prescribed ratio a/b is located dependence of q on x_a.

During calculation according to the isochronal curves

$$\sigma = \frac{Ke_0^m}{1+at^\beta}$$

it is possible to use the following relationship/ratios:

$$\Phi(x) = \frac{\Phi_0(x)}{1+at^\beta}, \quad x_a = x_{a_0}(1+at^\beta)^{1/m}. \qquad (44.12)$$

Here $\Phi_0(x)$ and x_{a_0} are determined for a curved instantaneous deformation, $\Phi(x)$ and x_a - for isochronal curve, that corresponds to the torque/moment of time t.

Example 44.1. To determine the stressed state and an increase in the bore diameter of closed duct is direct following by the rapid application/appendix of the internal pressure q=3000 AT. Bore diameter 2a=20 mm, external 2b - 40 mm. Material of duct - steel EI-696, operating temperature of 800°C.

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Expression (44.10) let us record in the form

$$\Phi(x)=\frac{1}{\sqrt{3}}\int_{0}^{x'}\frac{\varphi(x)}{x}dx+\frac{1}{\sqrt{3}}\int_{x'}^{x}\frac{\varphi(x)}{x}dx.$$

The first integral corresponds to the elastic part of the deformation, when $x \leqslant x'$. For steel EI-696 with T=800°C $x'=0.40\cdot10^{-2}$, $E=10^4$ kg/mm². Therefore for $x \leqslant 0.40\cdot10^{-2}$

$$\Phi(x)=\frac{Ex}{\sqrt{3}}\approx 5{,}77\cdot10^{3}x \text{ kg/mm}^2,$$

$\Phi(0.40\cdot10^{-2})=23{,}1$ kg/mm². For $x>0.40\cdot10^{-2}$ the function

$$\Phi(x)=23{,}1+\frac{1}{\sqrt{3}}\int_{0.40\cdot10^{-2}}^{x}\frac{\varphi(x)}{x}dx$$

is determined via numerical integration, value $\varphi(x)$ they are taken from the curved instantaneous deformation of Fig. 110. Graph for $\Phi(x)$ is constructed in Fig. 84.

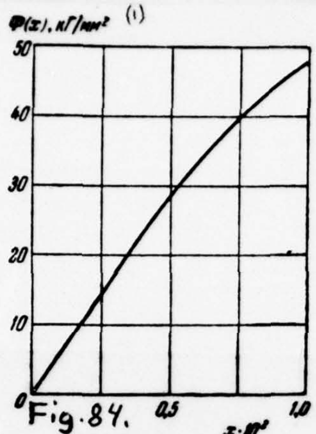

Fig.84.

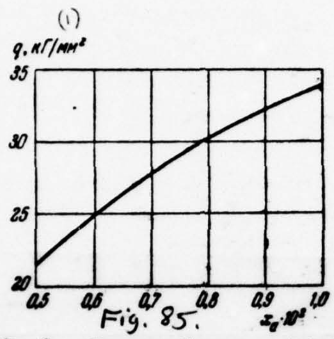

Fig. 85.

Fig. 84. To example of 44.1. Dependence $\Phi(x)$

Key: kg/mm².

Fig. 85. To example of 44.1. Dependence $q\ (x_a)$

Key: kg/mm².

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We utilize now formula (44.11) for finding dependence $x_a=C/a^2$ on the internal pressure q. Being assigned by the series of values x_a (from ~0,5•10⁻² to ~1,0•10⁻²), we search for appropriate values $x_b=\frac{a^2}{b^2}x_a$ and, after determining by the curve/graph of Fig. 84 values $\Phi(x_a)$ and $\Phi(x_b)$, let us find $q=\Phi(x_a)-\Phi(x_b)$. The obtained dependence is given in Fig. 85. For q=30 kg/mm² we have $x_a=0{,}790\cdot10^{-2}$.

Radial stresses are determined from formula (44.5):

$$\sigma_r=-q+\Phi(x_a)-\Phi(x)=11{,}4-\Phi\left(\frac{0{,}790}{r^2}\right).(*)$$

Circumferential stresses - on formula (44.7):

$$\sigma_\theta=\frac{2}{\sqrt{3}}\varphi\left(\frac{0{,}790}{r^2}\right)+\sigma_r.(**)$$

Obtained formulas (*), (**) each stress distribution according to a radius of duct is represented in Fig. 86 (solid lines).

An increase in the bore diameter will be according to formula (44.8)

$$\delta=\sqrt{3}ax_a=1{,}732\cdot10\cdot0{,}790\cdot10^{-2}=0{,}137\text{ mm}.$$

Example of 44.2. To determine the stressed state and an increase in the course of time in the bore diameter of copper circular cylindrical container. The initial value of bore diameter 2a=10 mm, external 2b=16 mm. Pressure within container 200 AT, temperature of 600°.

We take the equation of the curve of the instantaneous deformation of copper M1 in the form (see Fig. 130),

$$\varphi(e)=Ke^{m}. \qquad (*)$$

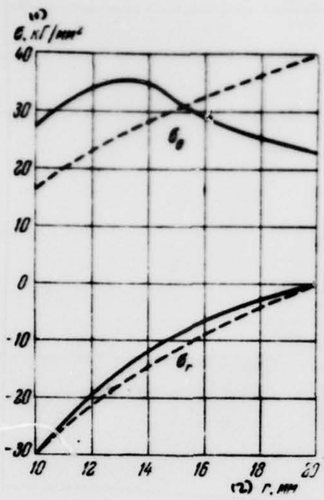

Fig. 86. Tc example of 44.1. Distribution cf voltages σ_r and σ_θ according to a radius of duct.

Key: (1). kg/mm². (2). mm.

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At 600°C K=22,8 kg/mm², m=0,457. Then

$$\Phi(x) = \frac{1}{\sqrt{3}} \int_0^x \frac{\varphi(x)}{x} dx = \frac{K}{\sqrt{3}m} x^m,$$

$$q = \Phi(x_a) - \Phi\left(x_a \frac{a^2}{b^2}\right) = \frac{K}{m\sqrt{3}} x_a^m \left[1 - \left(\frac{a}{b}\right)^{2m}\right].$$

From last/latter equation we will cbtain

$$x_a = \left[\frac{m\sqrt{3}}{K} \frac{q}{1 - \left(\frac{a}{b}\right)^{2m}}\right]^{1/m}. \qquad (**)$$

Consequently,

$$x = \frac{a^2}{r^2} x_a = \frac{a^2}{r^2} \left[\frac{m\sqrt{3}}{K} \frac{q}{1-\left(\frac{a}{b}\right)^{2m}} \right]^{1/m},$$

$$\varphi(x) = q \frac{m\sqrt{3}\left(\frac{a}{r}\right)^{2m}}{1-\left(\frac{a}{b}\right)^{2m}},$$

$$\Phi(x) = q \frac{\left(\frac{a}{r}\right)^{2m}}{1-\left(\frac{a}{b}\right)^{2m}}.$$

Expression (44.5) for radial displacement takes the form

$$\sigma_r = -q + \Phi(x_a) - \Phi(x) = -q \left[1 - \frac{1-\left(\frac{a}{r}\right)^{2m}}{1-\left(\frac{a}{b}\right)^{2m}} \right].$$

Circumferential stress

$$\sigma_\theta = \frac{2}{\sqrt{3}} \varphi(x) + \sigma_r = q \left[\frac{1-(1-2m)\left(\frac{a}{r}\right)^{2m}}{1-\left(\frac{a}{b}\right)^{2m}} - 1 \right].$$

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In the obtained expressions for voltages, value K does not enter. This means that if the isochronal curves are similar and are described by exponential function (*), then the stressed state of duct in the process of creep is not changed, if it are are determined according to the theory of aging.

The diagrams of voltages, calculated on the given formulas, are

constructed in Fig. 87.

Value x_a depending on time can be found from the formula

$$x_a = x_{a_0}(1 + at^{\beta})^{1/m},$$

where x_{a_0} is determined by expression (**). Consequently,

$$u(a) = \frac{\sqrt{3}}{2} a x_{a_0}(1 + at^{\beta})^{1/m}.$$

The calculated by this formula with a=0,110 (see Fig. 131) curve of an increase in the bore diameter of container is given in the course of time to Fig. 88.

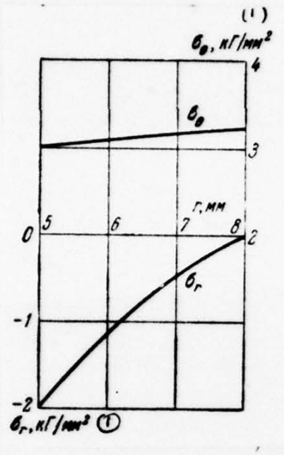

Fig 87

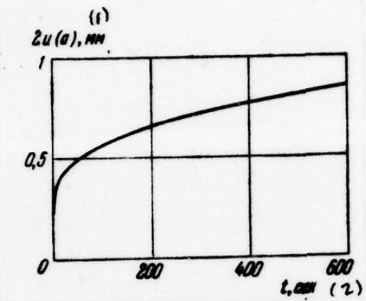

Fig. 88

Fig. 87. Tc example of 44.2. Distribution of vcltages σ_r and σ_θ according to a radius of duct.

Key: (1). kg/mm².

Fig. 88. Tc example of 44.2. Change in tube bore in time.

Key: (1). mm. (2). s.

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§45. The steady-state creep of thick-walled duct. Power law.

If the temperature differential according to the wall thickness of duct is not too great and index n in the power law of creep can be

accepted by constant, sclving of the task of steady-state creep actually does not differ frcm sclving of examined in preceding/previous paragraph task cf the plastic state of duct, only instead of displacement u it is necessary tc introduce radial velocity $\dot{u}$, instead of strains e_r and e_θ - rate of deformations ε_r and ε_θ. instead of the function $\Phi(e)$, that gives the dependence cf voltage from instantaneous deformaticn - function s(e), that is the voltage, which corresponds to creep rate ε. This function is taken now in the special form:

$$s(\varepsilon)=\sigma_n(r)\left(\frac{\varepsilon}{\varepsilon_n}\right)^{1/n}.$$

Without repeating all unpacking/facings, let us record formula for radial stress σ_r:

$$\sigma_r=-q+\frac{2}{\sqrt{3}}\int_a^r \sigma_n(r)\left(\frac{C}{\varepsilon_n r^2}\right)^{1/n}\frac{dr}{r}.$$

Let us accept for argument variable $\xi=a^2/r^2$, then formula for σ_r is rewritten as follows:

$$\sigma_r=-q-\frac{\mu}{\sqrt{3}}\int_1^\xi \sigma_n(\xi)\,\xi^{\frac{1}{n}-1}\,d\xi. \qquad (45.1)$$

After this it is easy tc find

$$\sigma_\theta=-q-\frac{\mu}{\sqrt{3}}\left[2\sigma_n(\xi)\,\xi^{1/n}+\int_1^\xi \sigma_n(\xi)\,\xi^{\frac{1}{n}-1}\,d\xi\right]. \qquad (45.2)$$

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Through μ is designated the following constant:

$$\mu=\left(\frac{C}{\varepsilon_n a^2}\right)^{1/n},$$

it is found from the second boundary condition for σ_r, and by nominal:

$$\mu = -\frac{q\sqrt{3}}{\int_1^{\beta^2} \sigma_n(\xi)\,\xi^{\frac{1}{n}-1}\,d\xi}. \qquad (45.3)$$

Here $\beta = b/a$.

Now we can calculate rate of change in the inside radius of the duct:

$$\dot{u}(a) = \frac{\sqrt{3}}{2}\,\varepsilon_n \mu^n a. \qquad (45.4)$$

If σ_n is constant, then all formulas are extracted in the closed form, namely:

$$\left.\begin{aligned} \sigma_r &= \frac{q}{\beta^{2/n}-1}\left[1-\left(\frac{b}{r}\right)^{2/n}\right], \\ \sigma_\theta &= \frac{q}{\beta^{2/n}-1}\left[1-\left(1-\frac{2}{n}\right)\left(\frac{b}{r}\right)^{2/n}\right]. \end{aligned}\right\} \qquad (45.5)$$

Now

$$\dot{u}(a) = \frac{\sqrt{3}}{2}\,\varepsilon_n a\beta^2\left[\frac{q\sqrt{3}}{n\sigma_n(\beta^{2/n}-1)}\right]^n. \qquad (45.6)$$

Example of 45.1. To find the stress distribution in the duct, examined in example of 44.1 in the state of steady-state creep. To determine the rate of an increase in the bore diameter.

The values of the necessary constants for steel EI-696 at temperature of 800°C will be: n=9,20, $\sigma_n = 26{,}0$ kg/mm², the domain of definition of constants $14<\sigma<40$ kg/mm².

Direct application/use of formulas (45.5) gives stress distribution, indicated on Fig. 86 by dashed lines.

The rate of an increase of the bore diameter in the state of steady-state creep will be

$$2\dot{u}(a) = \sqrt{3}\,\varepsilon_n a \beta^2 \left[\frac{q\sqrt{3}}{n\sigma_n(\beta^{2/n}-1)}\right]^n = \\ = \sqrt{3}\cdot 10^{-4}\cdot 10\cdot 2^2\left[\frac{30\sqrt{3}}{9{,}20\cdot 26{,}0\,(2^{2/9{,}70}-1)}\right]^{9{,}20} = \\ = 0.0977 \text{ mm/s}.$$

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§46. Approximate method of the calculation of ducts to creep.

If the temperature differential according to the wall thickness of duct is great and index n cannot be considered constant, the determination of exact solution by the method described above becomes difficult and it is impossible to indicate simple method for this. If the values of index n are sufficiently high, then stress distribution according to thickness differs little from that corresponding to the limiting condition of the duct for which the yield point at each point is proportional to value σ_n. Thus, we let us assume that the

stress intensity at each point exists $\lambda\sigma_n$, where λ - constant factor. On formula (44.3)

$$\sigma_\theta - \sigma_r = \frac{2}{\sqrt{3}}\lambda\sigma_n.$$

Let us introduce this expression into the equation of equilibrium and will integrate it. We will obtain

$$\sigma_r = -q + \frac{2}{\sqrt{3}}\lambda\int_1^{\rho}\frac{\sigma_n(\rho)}{\rho}d\rho. \qquad (46.1)$$

Here through ρ let us designate a dimensionless radius of duct. From boundary condition $\sigma_r(\beta)=0$, β=b/a, we obtain

$$\lambda = \frac{q\sqrt{3}}{2\int_1^{\beta}\frac{\sigma_n}{\rho}d\rho} = \frac{q}{q_*}.$$

Here

$$q_* = \frac{2}{\sqrt{3}}\int_1^{\beta}\frac{\sigma_n}{\rho}d\rho.$$

Let us calculate now the potential of creep for a duct according to the formula

$$\tilde{\Phi} = 2\pi\int_a^b\frac{e_n\sigma_n}{n+1}\left(\frac{\sigma}{\sigma_n}\right)^{n+1}r\,dr = 2\pi a^2 e_n\int_1^{\beta}\frac{\sigma_n}{n+1}\lambda^{n+1}\rho\,d\rho.$$

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We assume that the power, developed with the external load q, is equal to $2\pi a\dot{u}(a)q$; consequently, the rate of the generalized displacement, which corresponds to generalized force of q, exists $2\pi a u(a)$. This rate according to the Castigliano theorem is equal to derivative $\partial\tilde{\Phi}/\partial q$. Differentiating expression $\tilde{\Phi}$ from parameter q

under integral sign, we will obtain

$$\dot{u}(a)=\frac{\varepsilon_n a}{q_*}\int_1^\beta \sigma_n\left(\frac{q}{q_*}\right)^n \rho\, d\rho. \qquad (46.2)$$

In order to check approximation formula (46.2), it is logical to use it to any simple example for which there is exact solution. If n=const and $\sigma_n = \text{const}$, then

$$q_*=\frac{2}{\sqrt{3}}\sigma_n \ln\beta, \quad \int_1^\beta \sigma_n\left(\frac{q}{q_*}\right)^n \rho\, d\rho=\frac{\beta^2-1}{2}\left(\frac{q}{q_*}\right)^n \sigma_n$$

and

$$\dot{u}(a)=\frac{\sqrt{3}}{4}\varepsilon_n a\frac{\beta^2-1}{\ln\beta}\left(\frac{q\sqrt{3}}{2\sigma_n \ln\beta}\right)^n. \qquad (46.3)$$

This formula must be compared with (45.6). Let us first of all examine the case of thin-walled tube. Let us assume b-a=h and will consider that h/a≪1. Then, holding down only the first degrees of ratio h/a, we find that both of formula (45.6) and from (46.3) it follows:

$$\dot{u}(a)=\frac{\sqrt{3}}{2}\varepsilon_n a\left(\frac{qa\sqrt{3}}{2\sigma_n h}\right)^n. \qquad (46.4)$$

For drawing a comparison of the precise and approximation formula when wall thickness cannot be considered small, let us rewrite them in the form

$$\dot{u}(a)=\frac{\sqrt{3}}{2}\varepsilon_n a\left(\frac{q}{q_n}\right)^n. \qquad (46.5)$$

On the precise formula

$$q_n=\frac{n}{\sqrt{3}}\sigma_n\left[1-\left(\frac{a}{b}\right)^{2/n}\right]. \qquad (46.6)$$

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Cn the approximation formula

$$q_n = q_*\left(\frac{2\ln\beta}{\beta^2-1}\right)^{1/n} = \frac{2\sigma_n}{\sqrt{3}}\ln\beta\left(\frac{2\ln\beta}{\beta^2-1}\right)^{1/n}. \quad (46.7)$$

Figure 89 depicts the dependence cf value q_n/σ_n from index n in ratio b/a=2 on formulas (46.6) (solid line) and (46.7) (dashed line).

Example of 46.1. Thick-walled closed duct made of steel EI-696 is heated so that the temperature cf its internal wall $T_a = 800°$, external $T_b = 1000°$. Size/dimensicns of the duct: 2a=40 mm, 2b=60 mm. Internal pressure q=500 AT. It is required to determine the rate of an increase of the bore diameter in the state cf steady-state creep.

Considering the coefficient of thermal conductivity λ for steel EI-696 in the range of 80C-1000°C constant, let us record expression for a stationary temperature field in the cylindrical wall:

$$T(r) = T(a) + \frac{\Phi}{2\pi\lambda}\ln\frac{r}{a}.$$

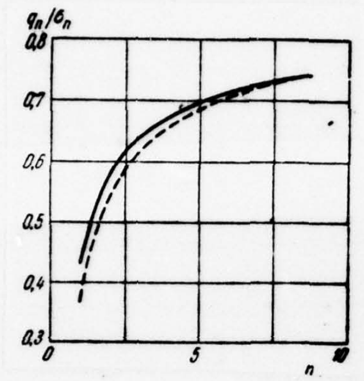

Fig. 89. Dependence of relation q_n/σ_n on n with b/a=2 on formulas (46.6) and (46.7).

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Here ψ - heat flow from an external radius of duct to internal. Since are known temperatures T_a and T_b, then

$$T(r) = T_a + (T_b - T_a)\frac{\ln \frac{r}{a}}{\ln \frac{b}{a}}.$$

The temperature distribution on this dependence is given in Fig. 90. Here are given to dependence σ_n and n on a radius in accordance with temperature field.

We determine value q_* by numerical integration for of these Fig. 90:

$$q_* = \frac{2}{\sqrt{3}} \int_1^{1,5} \frac{\sigma_n(\rho)}{\rho}\, d\rho = 4{,}92 \text{ kg/mm}^2.$$

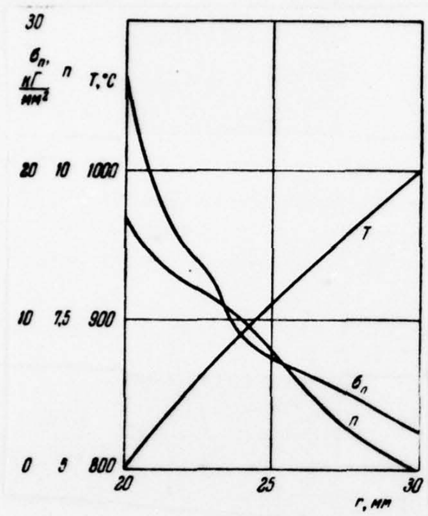

Fig. 90. To example of 46.1. Change T, n and σ_n in a radius of duct.

Key: (1). kg/mm². (2). mm.

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Rate of change the bore diameter will be

$$\delta(a) = \frac{2e_n a}{q_*} \int_1^{1,5} \sigma_n \left(\frac{q}{q_*}\right)^n \rho \, d\rho.$$

This integral is also calculated by numerical integration and in summation, is obtained

$\delta(a) = 0{,}0532$ by mm/s.

§47. Decomposition of thick-walled duct.

Let us examine the first closed evenly heated thick-walled duct. During steady-state creep the stressed state of this duct is described by formulas (45.5). Greatest normal stress is tangential stress σ_θ, the intensity of the voltages

$$\sigma_{\circ}=\frac{\sqrt{3}}{2}(\sigma_\theta-\sigma_r).$$

The value of equivalent voltage will be

$$\sigma_{э}=\frac{\sigma_\theta+\sigma_{\circ}}{2}=\frac{q}{2(\beta^{2/n}-1)}\left[1+\left(\frac{2+\sqrt{3}}{n}-1\right)\left(\frac{r}{b}\right)^{2/n}\right]. \quad (47.1)$$

With $n<2+\sqrt{3}$ value $\sigma_э$ reaches the greatest value on an inside radius of duct, whence begins decomposition. At $n>2+\sqrt{3}$ maximum $\sigma_э$ is reached on an outside radius. If $n=2+\sqrt{3}$, then in the state of steady-state creep the value of the equivalent voltage is constant according to the thickness of duct, the process of decomposition continues without the redistribution of voltages. Determining the stress-rupture strength of duct with $n\neq2+\sqrt{3}$ on maximum (47.1), we will obtain the lower estimation of bearing capacity.

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The maximum stressed state, which gives upper estimation, must

satisfy the equation of equilibrium (44.4)

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}=0,$$

to boundary conditions $\sigma_r=0$ with r=b, $\sigma_r=-q$ with r=a and to the condition

$$\sigma_э=\frac{\sigma_\theta+\sigma_\circ}{2}=\text{const}. \qquad (47.2)$$

Accepting the assumptions, formulated above (§44) which are given, in particular, to the relationship/ratio

$$\sigma_\circ=\frac{\sqrt{3}}{2}(\sigma_\theta-\sigma_r),$$

we will obtain, by solving the equation of the equilibrium:

$$\sigma_э=q\frac{1}{(\beta^{0,536}-1)^2}. \qquad (47.3)$$

Structural strength of duct is determined by relation $q/\sigma_э$. Therefore Fig. 91 depicts tc dependence (47.1) (with n=3) and (47.3) in coordinates $q/\sigma_э-\beta$. Index ' is related to lower estimation, index "- to upper. Lower estimation depends on value n, upper - it does not depend on n. At value n=2/√3, upper and lower estimations coincide.

If temperature is changed on a radius of duct, then $\sigma_э$, that corresponds to the assigned/prescribed service life, it is necessary to consider the function of a radius.

Let us examine also the case of the sufficiently fine/thin circular plate for which it is possible to accept $\sigma_z=0$. The equation of equilibrium and boundary conditions remain previous, equivalent voltage will be

$$\sigma_э=\frac{\sigma_\theta}{2}+\frac{1}{2}\sqrt{\sigma_\theta^2+\sigma_r^2-\sigma_\theta\sigma_r}. \qquad (47.4)$$

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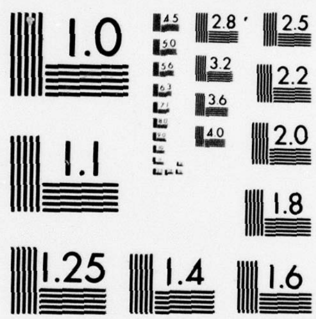

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Expressing σ_θ through σ_r and $\sigma_э$ with the aid of (47.4) and substituting the obtained expression in (44.4), we will obtain

$$\frac{4\sigma_э-\sigma_r}{4\left(\sigma_э^2-\sigma_э\sigma_r\right)}d\sigma_r=\frac{dr}{r}. \tag{47.5}$$

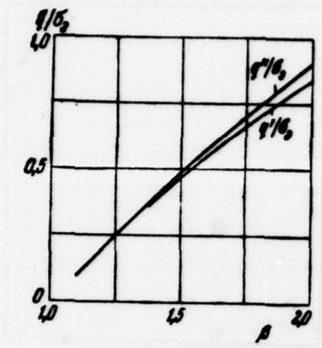

Fig. 91. Dependences q/σ_0 on β cn formulas (47.1) and (47.3).

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The solution of equation (47.5) under commcn boundary conditions it will be

$$\beta = \left(1 + \frac{q''}{\sigma_0}\right)^{3/4} \exp \frac{q''}{4\sigma_0}. \qquad (47.6)$$

If temperature is changed on a radius of plate, then equation (44.4) with the aid of conditicns (47.4) and (47.6) is reduced to following:

$$\sigma_r = -\int_r^b \frac{4\sigma_0^2(r, t_*) - \sigma_r^2}{(4\sigma_0(r, t_*) - \sigma_r)} \frac{dr}{r}. \qquad (47.7)$$

Its solution searches for by the method of successive approximations.

Example of 47.1. Tc determine the service life of the duct, examined in examples of 44.1 and 45.1.

The equivalent voltage for this duct in limiting condition will be

$$\sigma_{\text{э}} = q \frac{0{,}5}{\beta^{0{,}536} - 1} = 30 \frac{0{,}5}{2^{0{,}536} - 1} = 33{,}3 \text{ kg/mm}^2.$$

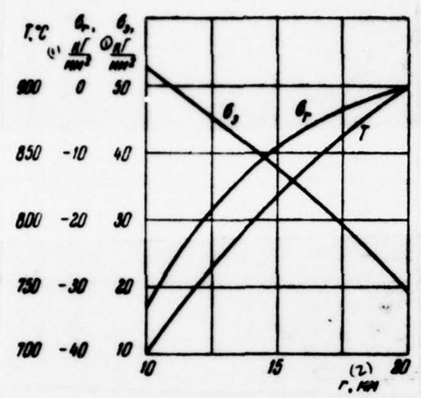

Fig. 92. To example of 47.2. Change **T,** σ_θ, **and** σ_r in a radius of duct.

Key: (1). kg/mm². (2). mm.

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To this voltage corresponds the upper estimation of service life $t_*'' = 79{,}5$ s. Lower estimation is obtained on the maximum voltage in the **state of** steady-state **creep. Maximum equivalent** voltage is reached on outside radius ($n > 2+\sqrt{3}$): $\sigma_e = \sigma_\theta(b) = 40{,}3$ kg/mm² (see example of 44.1). Consequently, $t_*'' = 10$ s.

Example of 47.2. To determine ultimate pressure in the circular plate, manufactured from the alloy EP-202; the inside radius a=10 mm,

external - b=20 mm: the temperature of internal wall $T_a = 700°C$, by C, external - $T_b = 900°$. Necessary service life of 1000 s.

The temperature field of plate is defined just as in example of 46.1, from the formula

$$T = T_a + (T_b - T_a)\frac{\ln\frac{r}{a}}{\ln\frac{b}{a}}.$$

The obtained dependence of temperature on a radius is given in table and in Fig. 92.

(1) r mm	T, °C	σ_θ	$-\sigma_r^{(0)}$	$-\sigma_r^{(1)}$	$-\sigma_r^{(2)}$	$-\sigma_r^{(3)}$	$-\sigma_r^{(4)}$
10	700	52,7	38,00	30,520	33,647	33,330	33,29
11	728	49,8	34,20	24,276	26,773	26,593	26,54
12	753	46,8	30,40	19,189	21,108	21,017	20,96
13	776	43,9	26,60	14,996	16,400	16,374	16,32
14	797	41,0	22,80	11,502	12,494	12,492	12,44
15	817	37,8	19,00	8,628	9,282	9,255	9,10
16	836	37,4	15,20	6,186	6,532	6,557	6,51
17	854	30,7	11,40	4,157	4,318	4,346	4,31
18	870	27,2	7,60	2,470	2,543	2,548	2,54
19	886	23,5	3,80	1,103	1,123	1,114	1,12
20	900	19,5	0	0	0	0	0

Key: (1). mm.

In Fig. 93 are plotted three experimental points of rupture stress of the alloy EP-202 for service life $t_* = 1000$ s and temperatures 700, 800 and 900°C, undertaken from Fig. 128.

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For determining intermediate values $\sigma_э$ is carried out interpolation curve. From this curved is remcved dependence $\sigma_э(r)$.

Distribution of racial stress is given by equation (47.7), which is solved by the method of successive approximations. As zero approximation let us take as

$$q^{(0)} = \sigma_э(\beta - 1),$$

counting temperature over the section/cut cf constant and equal to the temperature in point r=15 mm. Then $\sigma_э = 38{,}0$ kg/mm² and

$$q^{(0)} = 38{,}0 \text{ kg/mm}^2.$$

The distribution of radial stress $\sigma_r^{(0)}$ we take linear. The successive approximations, found frcm equation (47.9), are given in table (σ in kg/mm²).

As is evident, the third apprcach/apprcximation can be accepted as solving of task. The fcllowing approach/approximation more precisely formulates it insignificantly. Thus,

$$q'' = 33{,}3 \text{ kg/mm}^2.$$

Distribution of racial stress in limiting condition is given in Fig. 92.

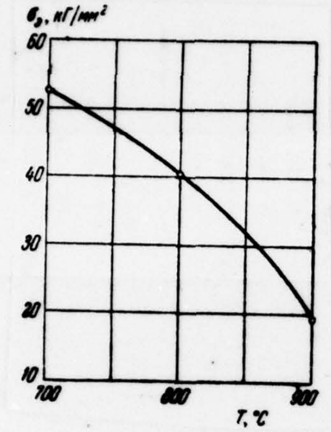

Fig. 93. Tc example of 47.2. Dependence $\sigma_в$ on T.

Key: (1). kg/mm².

End section.

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Chapter VI.

PROPERTIES OF STRUCTURAL MATERIALS.

§48. Preliminary observations.

As shown in the preceding/previous chapters, for the calculation of the elements of the construction/designs, subjected to rapid creep, it is necessary to have the following characteristics of materials:

1). the dependence of creep rate on voltage and temperature,

2). the curves of instantaneous deformation at different temperatures,

3). fracture characteristic,

4). some thermophysical constants.

In this chapter will be given the characteristics for some widespread structural alloys, determined in essence according to authors' experiments. During the use of these data, one should take into consideration that the fact that they depend substantially on the chemical composition of melting, the conditions/mode of heat treatment, form of semi-finished product, time of holding of specimen/sample at testing temperature without the load and some other factors. It is necessary to keep in mind also the heterogeneity of properties over the section/cut of blank, the anisotropy of the properties of polycrystalline materials. The environment (atmosphere of air, the vacuum of different degree, neutral medium, agressive media) they can substantially affect the mechanical behavior solids.

In order to avoid the errors in the estimation of strength and rigidity of construction/design, it is necessary during the use of characteristics of materials to consider experimental conditions, in which they are obtained. These conditions, and also the characteristics of initial material as far as possible fully are indicated below for each investigated material.

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For the calculation of the heavy-duty/critical construction/designs, and also when it is necessary as it is possible to precisely estimate bearing capacity, one should obtain the characteristics of that material which directly is utilized for its manufacture, in that structural state, in which this material is utilized in construction/design. Determining technique the corresponding characteristics is comparatively simple and nonlaborious. Is given below the necessary information about procedure and experimental technique and the methods of processing experimental data.

§49. Determining the characteristics of rapid creep.

Rapid creep for the majority of materials occurs without noticeable strengthening. The deformation rate, in this case, does not depend on the prehistory of deformation. Therefore, creep rate is the function of voltage, which is approximated by expressions (14.1) and (14.3). The entering these equations parameters let us call the constants of creep. For the determination of these constants, was applied the following procedure.

The series of specimen/samples (three-five pieces) undergoes stepped loading at fixed/recorded temperature T_k, with the aid of the appropriate testing unit (its short description is contained in

appendix) the voltage in specimen/sample is changed with step/stages, so that single-stage duration composes time from several seconds to several hundred seconds (depending on creep rate) and transit time from one step/stage to another - 1-2 s. Exemplary/approximate program is shown on Fig. 94. At each step/stage of loading, is remove/taken the curve of the dependence of creep strain on time, from which is determined the creep rate.

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Thus, in each specimen/sample number j, tested at temperature T_k, is obtained sequence from q_j the points of diagram $\varepsilon-\sigma$.

If is accepted the law of creep (14.1), then these points, plotted/applied in coordinates $\lg\varepsilon-\sigma$, are determined straight line

$$\lg \varepsilon = \lg \varepsilon_{ej} + 0{,}434 \frac{\sigma}{\sigma_{ej}}.$$

Values ε_{ej} and σ_{ej} for each specimen/sample (number j) are located by the method of least squares. After this is manufactured the averaging of the obtained characteristics for different specimen/samples.

Completely analogously are located the constants of the power law of creep (14.3), to which corresponds the linear dependence of velocity on the voltage in the logarithmic coordinates:

$$\lg \frac{\varepsilon}{\varepsilon_n} = n \lg \sigma - n \lg \sigma_n.$$

After this value of the constants, obtained during testing of different specimen/samples, they are averaged in the usual way.

In formula (14.3) one of constants ε_n or σ_n can be fixed on arbitrariness. Subsequently it is everywhere accepted $\varepsilon_n = 10^{-4}$.

The constants of creep are the functions of temperature. Since all problems of creep in variable temperature field are solved numerically, there is no sense to select general purpose empirical formulas for the dependence of constants on temperature. These dependences are represented in the form of graphs in Fig. 13 and 14.

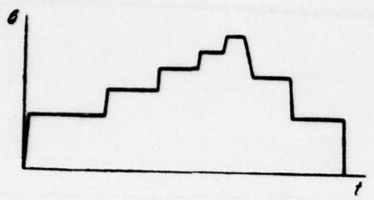

Fig. 94. Exemplary/approximate program of the loading of specimen/sample.

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In certain cases of the approximation of temperature dependences, they can all the same prove to be useful. In particular, for many materials are convenient the following approximations for the values, entering the exponential law of creep:

$$\sigma_e = \text{const}, \qquad \varepsilon_e = \varepsilon_{e1} \exp \nu_1 T \qquad (49.1)$$

for low temperatures and

$$\sigma_e = \frac{T_0 - T}{\beta}, \qquad \varepsilon_e = \varepsilon_{e2} \exp \nu_2 T \qquad (49.2)$$

for high temperatures.

Here $\varepsilon_{e_1}, \nu_1, T_0, \beta, \nu_2, \varepsilon_{e_2}$ - constant.

For solving the separate specific problems, it is possible to recommend to construct most convenient for these tasks temperature

dependences, by determining them, perhaps, in the narrow temperature range, necessary for the task in question. In this way it is possible to simplify solution and to obtain high accuracy/precision. Examples of this type were given above (for example, intc §23).

When it is not possible tc disregard the unsteady section of curve of creep, it should be used the isochrcnal curves construction technique of which is described abcve (§16). There it is noted, that the family of isochronal curves is usually described well by the formula

$$\sigma=\frac{\varphi(e)}{1+at^{\beta}},$$

where ϕ(e) - curve of instantaneous deformation, a=a(T) and β=const.

For determining entire totality of isochrcnal curves in the assigned/prescribed temperature interval, are given below the values β, is assigned graphically function a(T) and the grid of the curves cf instantaneous deformation. These data make it possible by interpclation to obtain I admire isochronal curve for the assigned/prescribed temperature.

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§50. Curves of instantaneous deformation.

The procedure of the plotting of curves of instantaneous deformation was presented above (§5).

Experiments regarding the characteristics of instantaneous deformation are conducted during setting up at a high speed of loading (for example, during the setting up with electromagnetic actuator, described in appendix) at certain fixed/recorded temperatures with the sufficiently large intervals between them (50-100°). For convenience in the practical use, the difference of temperatures between the separate curves which are given below, comprises, as a rule, 25°. These curves are obtained by interpolation of experimental data. If necessary on the available curves, can be obtained the grid of the diagrams of instantaneous deformation with smaller step/pitch.

If for the calculation of the element of construction/design to creep is necessary the knowledge of the modulus of elasticity, then it should be taken on the curves of instantaneous deformation. Preferences the dynamic modulus in this case given should not be. The low accuracy/precision of the determination of the modulus of elasticity from the curve of instantaneous deformation introduces errors only into the determination of initial elastic state, which usually in pure form in the construction/designs in question is not realized, since loading is accompanied usually either by

instantaneous plastic strains or creep strains.

§51. Fracture characteristics during rapid creep.

It is possible to indicate three types of the decomposition, characteristic for the rapid creep:

1). Strain during decomposition does not depend on voltage $e_* = e_*(T)$. In this case is given below dependence $e_*(T)$, which uniquely determines the torque/moment of decomposition.

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2). Strain during decomposition depends on voltage $e_* = e_*(\sigma)$. For this case is given the so-called curve of stress-rupture strength - dependence of rupture stress from the time of its action (term reflects the time/temporary dependence of decomposition, but not the duration of process).

3). At sufficiently high temperature the decomposition during rapid creep has viscous character. (Among the investigated materials exception is pure copper). Therefore in given below reference data is indicated for each alloy the temperature, higher than which calculation by service life must be fulfilled without taking into account of embrittlement.

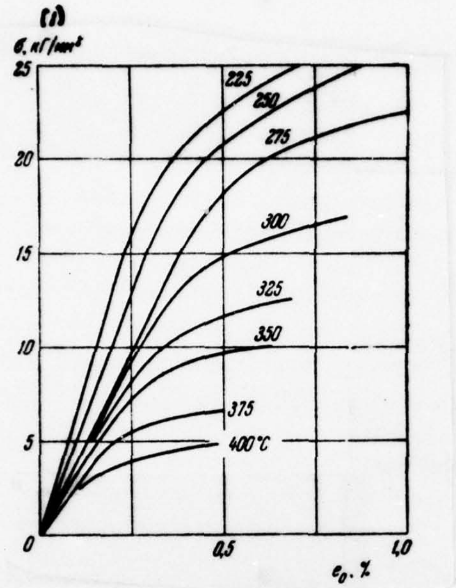

Fig. 95. D16AT, sheet 2 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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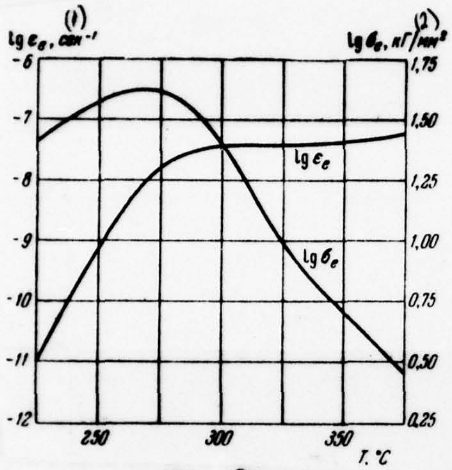

Fig. 96.

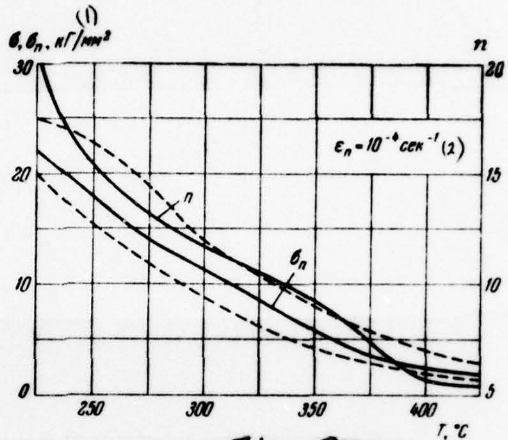

Fig. 97.

Fig. 96. D16AT, sheet 2 mm. Dependence of the constants of exponential law on temperature.

Key: (1). s^{-1}. (2). kg/mm^2.

Fig. 97. D16AT, sheet 2 mm. Dependence of the constants of power law on temperature.

Key: (1). kg/mm^2. (2). s^{-1}.

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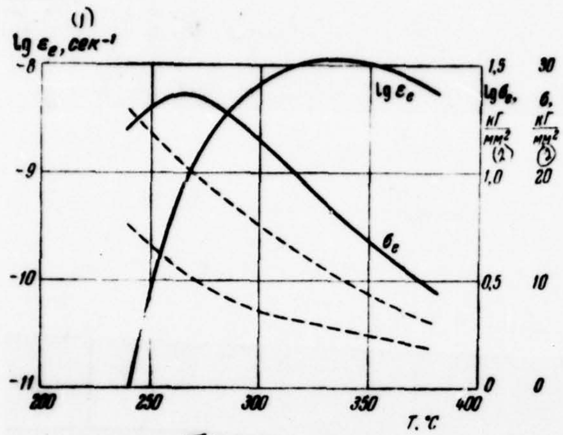

Fig. 98.

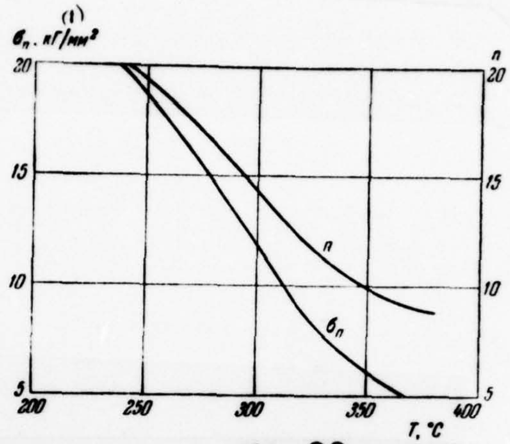

Fig. 99

Fig. 98. D16T, rod 30 mm. Dependence of the constants of exponential law on temperature.

Key: (1). s^{-1}. (2). kg/mm^2.

Fig. 99. D16T, rod 30 mm. Dependence of the constants of power law on temperature.

Key: (1). kg/mm^2.

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§52. Characteristics of some materials.

Aluminum alloy D16 (Fig. 95-99).

The brand chemical composition according tc GOST 4784-49: 3.8-4.9o/o Cu, 0.3-0.7o/o Mn, 1.2-1.6o/o Mg, ⩽0,5o/o Fe, ⩽0.3o/o Si, ⩽0,1o/o Ni.

In figures are given the characteristics, obtained for two forms cf the semi-finished prcduct:

1). Sheet 2 mm in thickness D16AT (harden/tempered and logically aged), hardness HRB=62-66, specimen/samples were cut out in direction cf rolling.

2). Rod 30 mm in diameter D16T (harden/tempered and logically aged).

The domains of definition of the constants of exponential and power laws for all examined materials coincide. Further for all materials they will be applied cnly on cne cf graphs $\lg e_{*},\ \sigma_e - T$ or $\sigma_n,\ n - T$ by dashed lines. In certain cases fcr convenience in the reading, the scale for voltages is accepted logarithmic.

T°C	240	270	330	380
e_*	0,246	0,236	0,257	0,443

Embrittlement during rapid creep for an alloy D16 can be

disregarded with T>380°C.

Titanium alloy OT-4 (Fig. 100-102).

The chemical composition: 2.0-3.5 (2.80)o/o Al, 1.0-2.0 (0.95)o/o Mn, ⩽0,4 (0.15)o/o Fe, ⩽0,1 (0.04)o/o, C, ⩽0,15o/o Si, ⩽0,05 (0.028)o/o N_2⩽0,15o/o O_2, ⩽0,015 (0.006)o/o H_2.

Is given the brand chemical composition by AMTU 475-3-61 and in brackets - melting, used for determining of the characteristics of rapid creep. Tested sheet 3 mm in thickness, specimen/samples were cut out in direction of rolling. Conditions/modes of annealing.

Conditions/modes of annealing.

Version I: heating in vacuum are not worse $5\cdot10^{-2}$ tor to 860°C during 4 hours of 45 min., holding 20 min., cooling in vacuum to 150° during 4 hour.

Version II: heating in vacuum is not worse $5\cdot10^{-2}$ tor to 825° during 4 hours of 45 min., holding 2 min. cooling in vacuum to 150° during 4 hour.

Hardness after annealing on version I HV30=307, after annealing according to Version II HV30=302.

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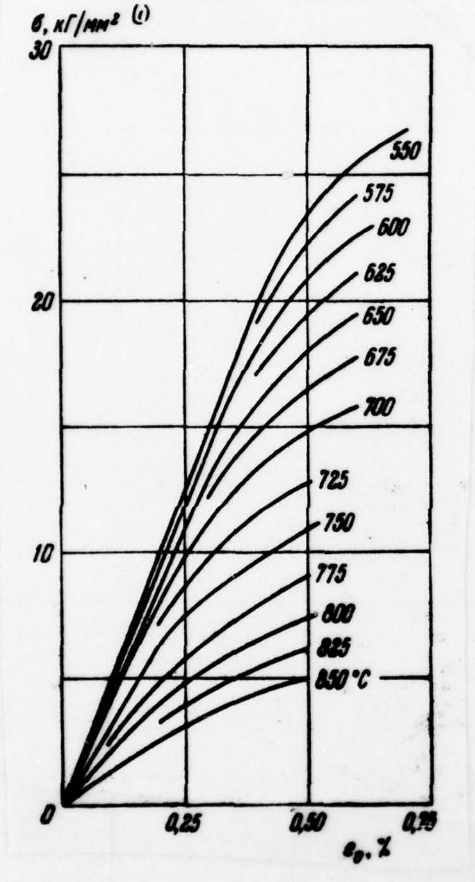

Fig. 100. OT-4, sheet 3 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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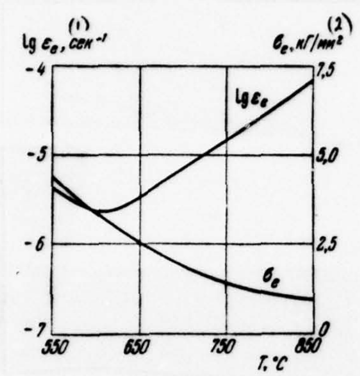

Fig. 101. OT-4, sheet 3 mm. Dependence of the constants of exponential law on temperature.

Key: (1). s^{-1}. (2). kg/mm^2.

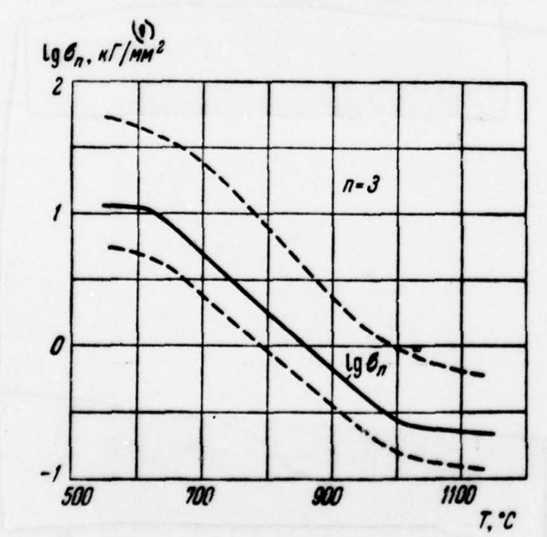

Fig. 102. OT-4, sheet 3 mm. Dependence of the constants of power law on temperature.

Key: (1). kg/mm².

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Mechanical characteristics at high temperatures virtually coincide for both versions of the conditions/mode of annealing; therefore are given below the averaged characteristics, suitable in both cases.

Embrittlement during rapid creep can be disregarded with T>700°C.

Titanium alloy VT-14 (Fig. 103-105).

The chemical composition by AMTU 471-61: 3,5-4,5o/o Al, 2,5-3,5o/o Mo, 0,7-1,5o/o V, $\leqslant$0,4o/o Fe, $\leqslant$0,15o/o Si, $\leqslant$0,1o/o C, $\leqslant$0,15o/o O_2, $\leqslant$0,015o/o H_2, $\leqslant$0,05o/o N_2.

Tested sheet 2 mm in thickness, specimen/samples were cut out in direction of rolling.

Conditions/mode of the heat treatment: annealing at temperature

of 850°C for 1 hour, cooling in air. Hardness after annealing HV 30=296-307.

Possible approximation of the temperature dependences of the parameters of exponential law in the range of temperatures of 600-800°C:

$$\sigma_e = \frac{T_0 - T}{\beta}, \qquad \varepsilon_e = \varepsilon_{e2} \exp \nu_2 T,$$
$$\varepsilon_{e2} = 1{,}95 \cdot 10^{-10}\ сек^{-1}, ^{(1)} \qquad T_0 = 896°\ \mathrm{C},$$
$$\nu_2 = 0{,}0141\ град^{-1}, ^{(2)} \qquad \beta = 70{,}4\ мм^2 град/кГ. ^{(3)}$$

Key: (1). s^{-1}. (2). deg^{-1}. (3). mm^2deg/kgf.

Embrittlement during rapid creep can be disregarded at temperatures T>500°C.

Stainless steel Kh18N10T (Fig. 106-109).

The chemical composition according to GOST 10994-64 (in brackets - investigated melting): ≤0.12(0.10)o/o C, 17-19(17.40)o/o Cr, 9-11(11.0)o/o Ni, to 0.7(0.5)o/o Ti, 1-2 (1,00o/o Mn, ≤0,8 (0.42)o/o Si, (0.011)o/o S, (0.024)o/o P.

Tested cold-rolled sheet 2.5 mm in thickness, specimen/samples were cut out in direction of rolling.

Conditions/mode of heat treatment - heating to temperature of 1070°C, holding during 20 min., cooling in air. Hardness after heat treatment HV30=136-144.

The curves of rapid creep after the heat treatment indicated up to temperature of 800°C are characterized by considerable strengthening. At the temperatures lower than 600° rapid creep (for a period of up to 1 hour) can be disregarded.

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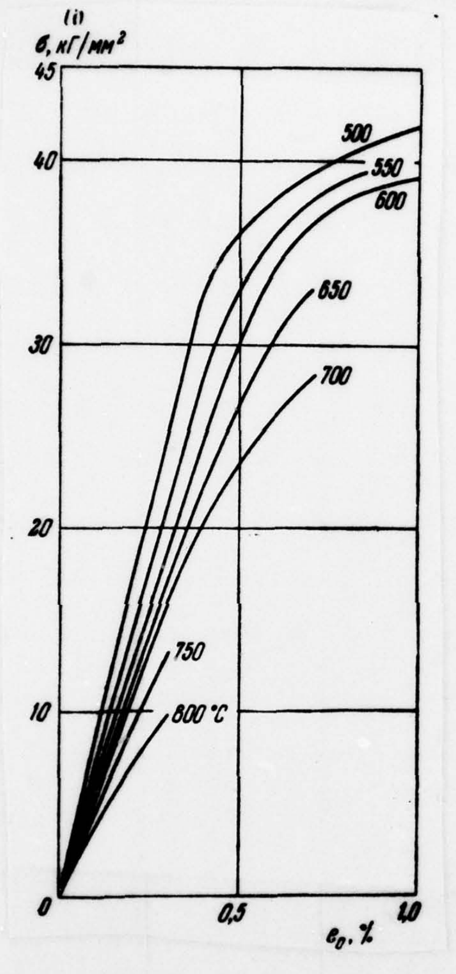

Fig. 103. VT-14, sheet 2 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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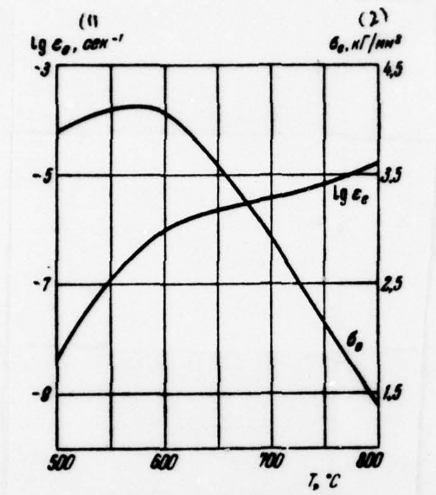

Fig. 104. VT-14, sheet 2 mm. Dependence of the constants of exponential law on temperature.

Key: (1). s⁻¹. (2). kg/mm².

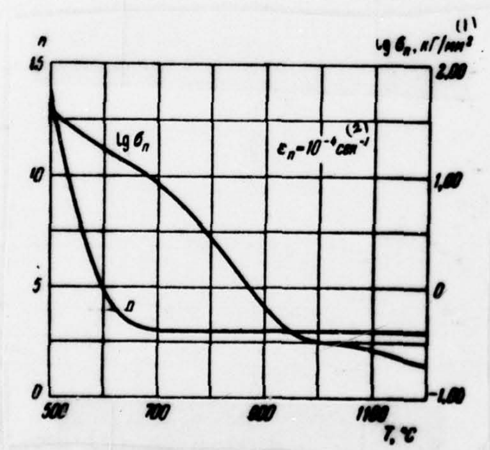

Fig. 105.

Fig. 105. VT-14, sheet 2 mm. Dependence of the constants of power law cn temperature.

Key: (1). kg/mm^2. (2). s^{-1}.

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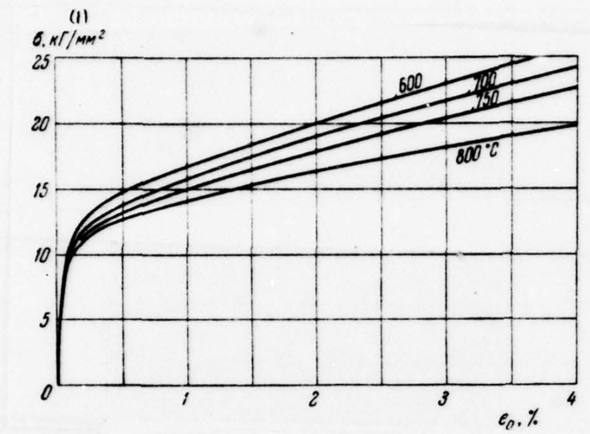

Fig. 106. Kh18N10T, sheet 2.5 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

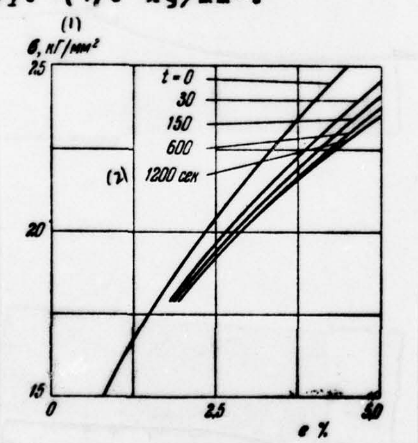

Fig. 107. Kh18N10T, sheet 2.5 mm, cf 700°C. Isochronal curves.

Key: (1). kg/mm². (2). s.

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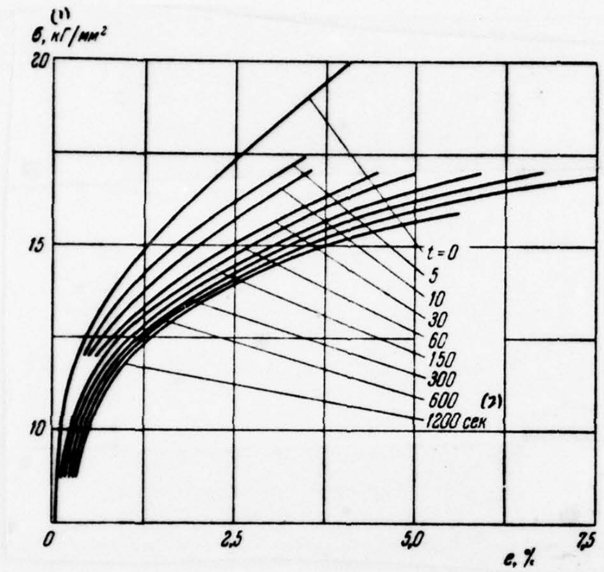

Fig. 108. Kh18N10T, sheet 2.5 mm, cf 800°C. Isochronal curves.

Key: (1). kg/mm². (2). s.

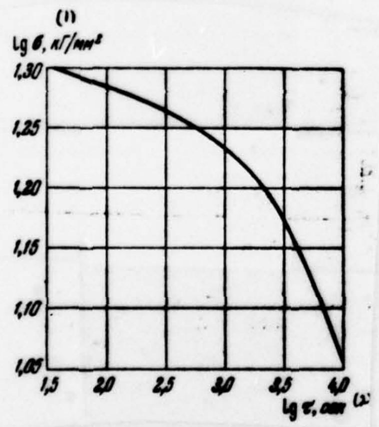

Fig. 109. Kh18N10T, sheet 2.5 mm, cf 800°C. Dependence of the time of decomposition on voltage.

Key: (1). kg/mm². (2). s.

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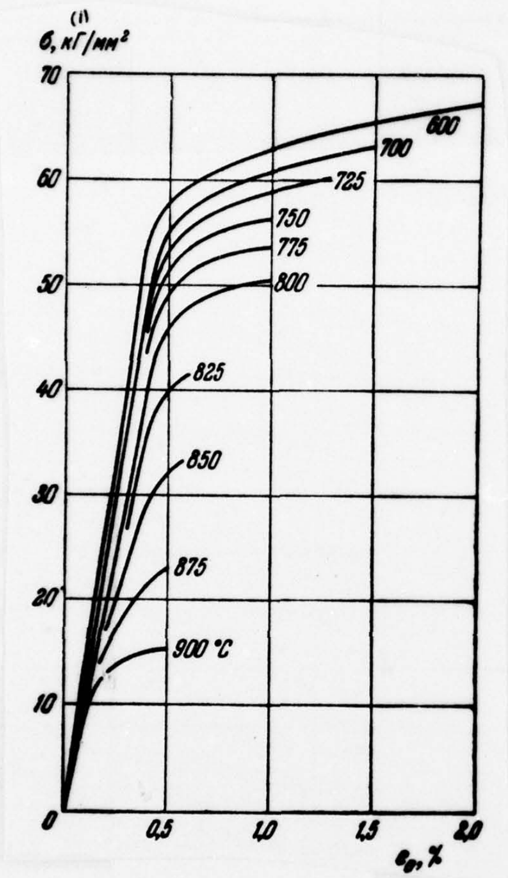

Fig. 110. EI-696, rod 40 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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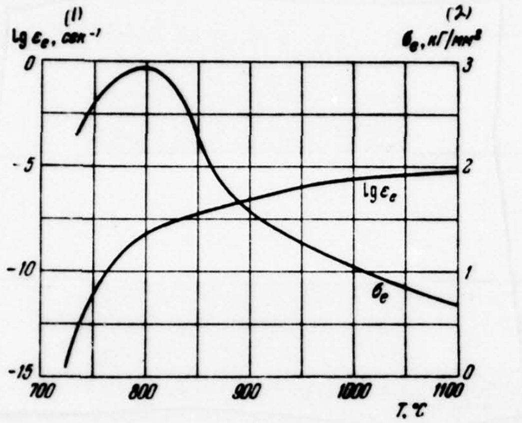

Fig. 111. EI-696, rod 40 mm. Dependence of the constants of exponential law on temperature.

Key: (1). s^{-1}. (2). kg/mm².

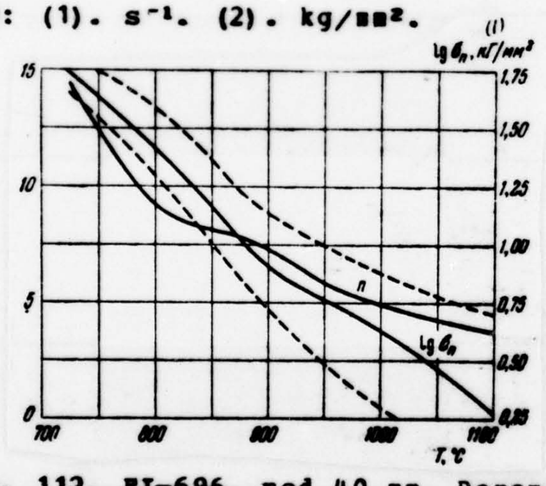

Fig. 112. EI-696, rod 40 mm. Dependence of the constants of power law on temperature.

Key: (1). kg/mm².

Eage 202.

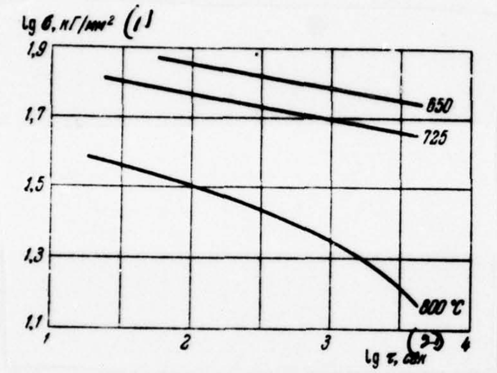

Fig. 113.

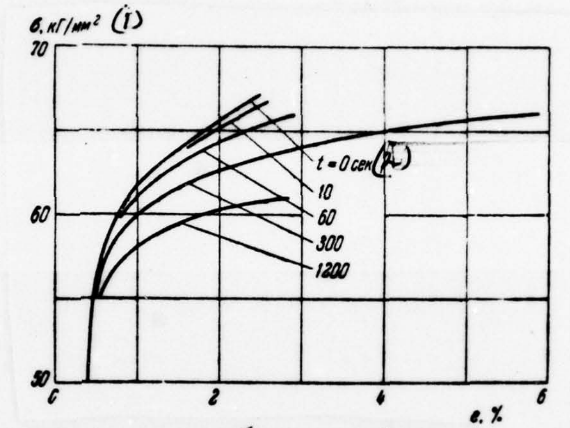

Fig. 114.

Fig. 113. EI-696, rod 40 mm. Dependence of the time of destruction on voltage/stress.

Key: (1). kg/mm². (2). s.

Fig. 114. EI-696, rod 40 mm, of 650°C. Isochronal curves.

Key: (1). kg/mm². (2). s.

Eage 203.

Heat-resistant steel EI-696 (Fig. 110-114).

The chemical composition according to GOST 10994-64 (in brackets - investigated melting): ≤0,10(0.05)o/o C, ≤1,0(0.37)o/o Si, ≤1,0(0.50)o/o Mn, 10.0-12.5(11.0)o/o Cr, 18.0-21.0 (19.63)o/o Ni, 2.6-3.2 (2.87)o/o Ti, ≤0,8(0.25)o/o Al, 0.008-0.2 (0.008)o/o B.

Tested rod 40 mm in diameter.

Conditions/mode of the heat treatment: tempering from 1150°C, the holding of 2 hours, cooling in air; aging of 760°C - 20 hour, cooling in air. Hardness after heat treatment HV30=329--338.

Possible approximations of the temperature dependences of the parameters of the exponential law:

$$\varepsilon_e = \varepsilon_{e1,2} \exp \nu T, \qquad \sigma_e = \frac{T_0 - T}{\beta}.$$

In the range of temperatures of 725-845°C:

$$\varepsilon_{e1} = 3{,}64 \cdot 10^{-48}\ сек^{-1},^{(1)} \quad \sigma_{e1} = \text{const} = 3{,}17\ кГ/мм^2,^{(3)}$$
$$\nu_1 = 0{,}1144\ град^{-1}.^{(2)}$$

Key: (1). s^{-1}. (2). deg^{-1}. (3). kg/mm^2.

In the range of temperatures of 870-1150°C.

$$\varepsilon_{e2} = 2{,}19 \cdot 10^{-14}\ сек^{-1},^{(1)} \quad T_0 = 1303^\circ C,^{(3)}$$
$$\nu_2 = 0{,}0178\ град^{-1},^{(2)} \quad \beta = 237\ мм^2 град/кГ.$$

Key: (1). s^{-1}. (2). deg^{-1}. mm^2deg/kgf.

Stainless steel EI-654 (Fig. 115-117).

Tested the specimen/samples, cut out made of rod 120 mm in diameter made of the zone of secticn/cut in bore 60 mm diameters and external 100 mm.

The conditions/mode of the heat treatment: 1) preheating in salt bath to 820°C, holding 10 min., 2) heating in barium bath to 1050°C, holding 5 min., cooling - water, 3) the repetition of the operation of the first, 4) the repetition of operation the second. Hardness after heat treatment HV30=243.

Possible approximaticns of the temperature dependences of the parameters of the exponential law:

$$e_e = e_{e1,2} \exp \nu T, \qquad \sigma_e = \frac{T_0 - T}{\beta}.$$

In the range of temperatures of 700-912°C:

$$e_{e1} = 4{,}17 \cdot 10^{-13}\ сек^{-1}, (1) \quad T_0 = 1035°\,C,$$
$$\nu_1 = 0{,}0148\ град^{-1}, (2) \quad \beta = 143\ мм^2град/кГ. (3)$$

Key: (1). s^{-1}. (2). deg^{-1}. (3). mm^2deg/kgf.

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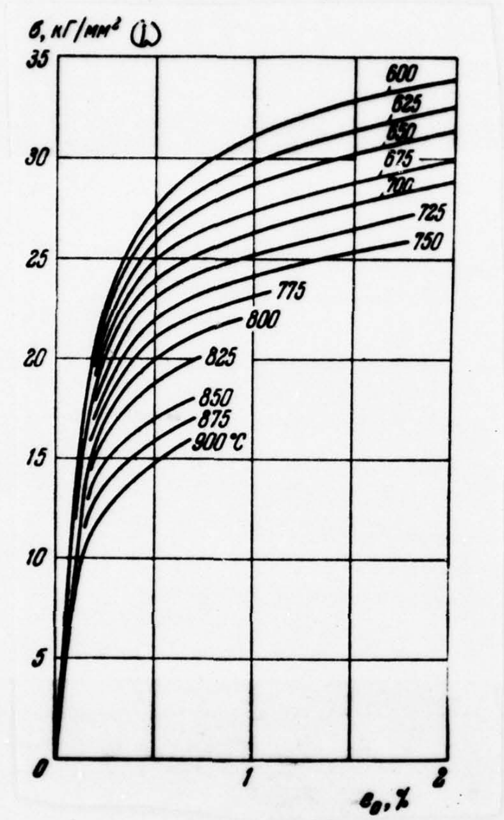

Fig. 115. EI-654, rod 120 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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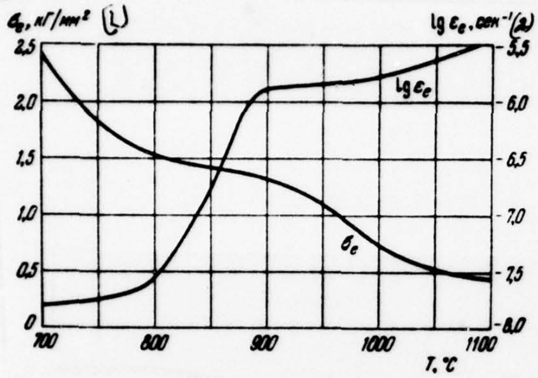

Fig. 116.

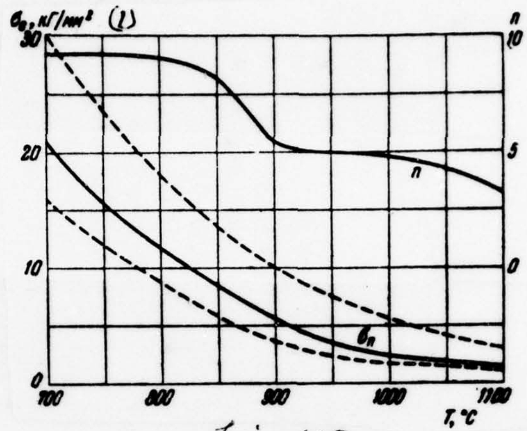

Fig. 117.

Fig. 116. EI-654, rod 120 mm. Dependence of the constants of exponential law on temperature.

Key: (1). kg/mm². (2). s⁻¹.

Fig. 117. EI-654, rod 120 mm. Dependence of the constants of power law on temperature.

Key: (1). kg/mm².

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In the range of temperatures of 912-1100°C:

$$\varepsilon_{e2} = 4{,}17 \cdot 10^{-13}\ сек^{-1},\ (1) \quad T_0 = 1328^\circ\ C,\ (3)$$
$$\nu_2 = 0{,}0148\ град^{-1},\ (2) \quad \beta = 478\ мм^2 град/кГ.$$

Key: (1). s^{-1}. (2). deg^{-1}. (3). $mm^2 deg/khf$.

Embrittlement during rapid creep it is possible to disregard at $600 \leq T \leq 1100°C$.

Stainless steel EI-811 (Fig. 118-121).

The chemical composition of the investigated melting: 0,9o/o C, 4,91o/o Ni, 21,25o/o Cr, 0,52o/o Ti, 0,52o/o Mn, 0,56o/o Si, 0,010o/o S, 0,025o/o P.

Tested the specimen/samples, cut out made of cold-rolled sheet 2 mm in thickness in direction of rolling.

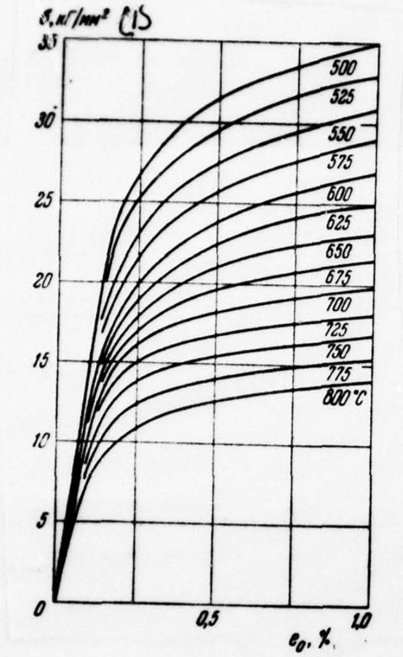

Fig. 118. EI-811, sheet 2 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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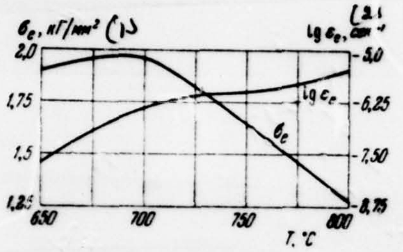

Fig. 119.

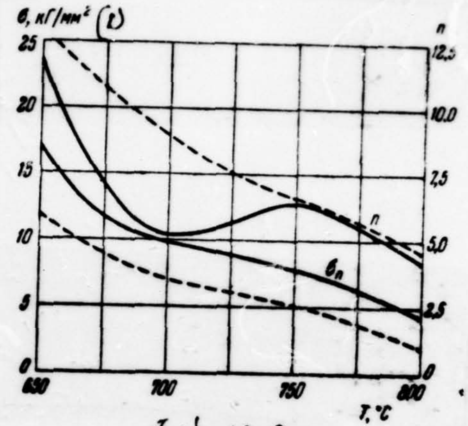

Fig. 120.

Fig. 119. EI-811, sheet 2 mm. Dependence of the constants of exponential law on temperature.

Key: (1). kg/mm². (2). s^{-1}.

Fig. 120. EI-811, sheet 2 mm. Dependence of the constants of power law on temperature.

Key: (1). kg/mm².

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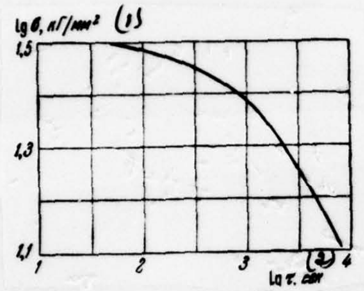

Fig. 121.

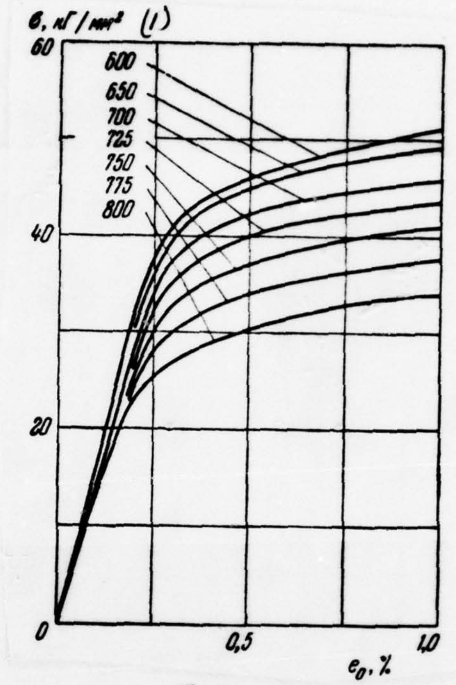

Fig. 122.

Fig. 121. EI-811, sheet 2 mm, of 600°C. Dependence of the time of destruction on voltage/stress.

Key: (1). kg/mm². (2). s.

Fig. 122. EP-164, rod 100 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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Conditions/mode of heat treatment - heating to the temperature 1000°C, holding for 30 min., cooling in water. Hardness after heat treatment HV30=205-208.

Possible approximations of the temperature dependences of the parameters of exponential law in the range of temperatures of 650-800°C:

$$\varepsilon_c = \varepsilon_{c2} \exp \nu T, \qquad \sigma_c = \frac{T_0 - T}{\beta},$$
$$\varepsilon_{c2} = 8{,}32 \cdot 10^{-17}\ \text{сек}^{-1},\ (1) \qquad T_0 = 1128^\circ \text{C},\ (3)$$
$$\nu = 0{,}0133\ \text{град}^{-1},\ (2) \qquad \beta = 240\ \text{мм}^2\text{град/кГ}.$$

Key: (1). s^{-1}. (2). deg^{-1}. (3). mm^2deg/kgf.

Stainless steel EP-164 (Fig. 122-124).

The chemical composition of the melting: 0.06o/o C, 23.84o/o Ni, 14.95o/o Cr, 1.76o/o Ti, 0.70o/o Mn, 0.19o/o Si, 0.008o/o S, 0.007o/o P, 0,005o/o B, 0.025o/o Ce.

Tested rod 100 mm in diameter. Conditions/mode of the heat treatment: 1) quenching from temperature of 1130°C, holding - 2 hours, coolings in water of the room temperature, 2) aging at temperature of 750°C during 16 hours, 3) aging at temperature of 700°C during 8 hours, cooling in air. Hardness after heat treatment HV30=270-304.

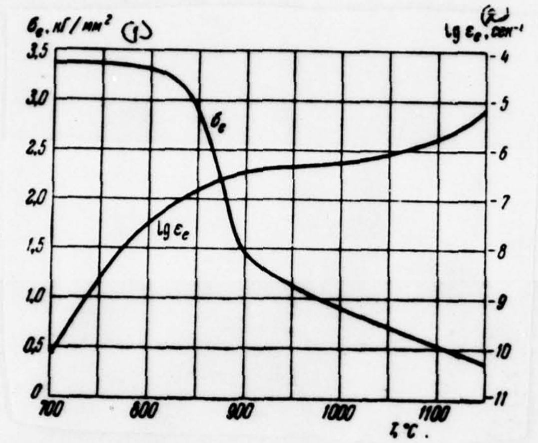

Fig. 123. EP-164, rod 100 mm. Dependence of the constants of exponential law on temperature.

Key: (1). kg/mm². (2). s⁻¹.

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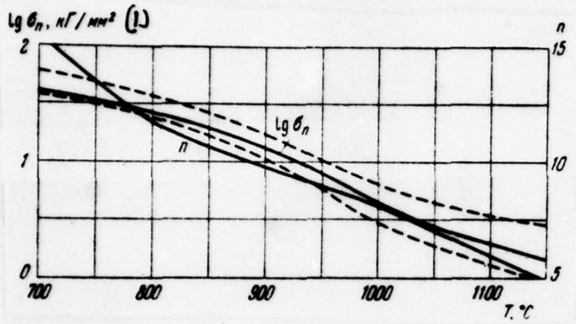

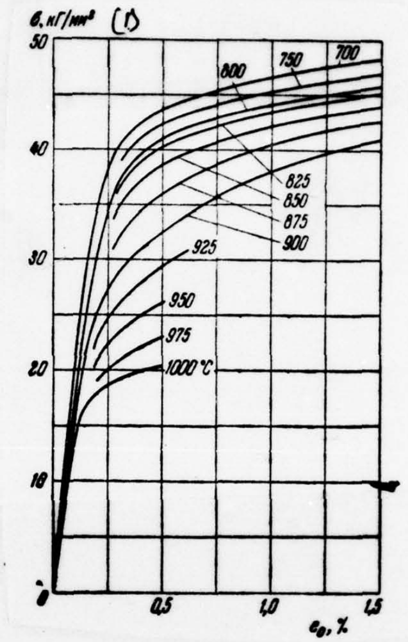

Fig. 124. EP-164, rod 100 mm. Dependence of the constants of power law on temperature.

Key: (1). kg/mm².

Fig. 125. EP-202, sheet 2.5 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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Possible approximations of the temperature dependences of the parameters of the exponential law:

$$\varepsilon_e = \varepsilon_{e1,2} \exp \nu T, \qquad \sigma_e = \frac{T_\circ - T}{\beta}.$$

In the range of temperatures 700-865°C.:

$$\varepsilon_{e1} = 3{,}90 \cdot 10^{-30}\ сек^{-1},\ (1) \qquad \sigma_{e1} = \mathrm{const} = 3{,}33\ кГ/мм^2,\ (2)$$
$$\nu_1 = 0{,}0633\ град^{-1}.\ (3)$$

Key: (1). s^{-1}. (2). kg/mm^2. (3). deg^{-1}.

In the range of temperatures of 910-1150°C:

$$\varepsilon_{e2} = 8{,}52 \cdot 10^{-12}\ сек^{-1},\ (1) \qquad T_\circ = 1255°\ C.$$
$$\nu_2 = 0{,}0116\ град^{-1},\ (2) \qquad \beta = 260\ мм^2 град/кГ.\ (3)$$

Key: (1). s^{-1}. (2). deg^{-1}. (3). $mm^2 deg/kgf$.

Nickel alloy EP-202 (Fig. 125-128).

The chemical composition of the melting: 18.95o/o Cr, 4.88o/o W, 4.73o/o Mo, 0.72o/o Fe, 2,46o/o Ti, 1.30o/o Al, 0,06o/o C, 0.26o/o Si, 0.005o/o B, 0.007o/o S, 0.009o/o P.

Tested sheet 2.5 mm in thickness, specimen/samples were cut out in direction of rolling.

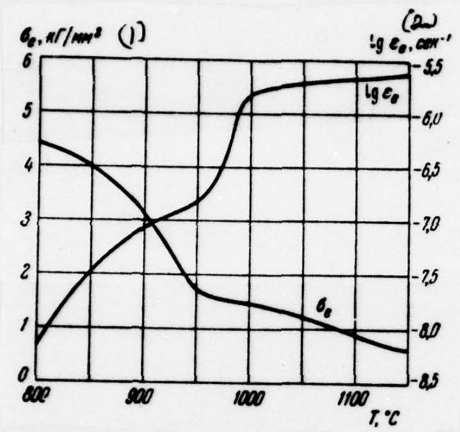

Fig. 126. EP-202, sheet 2.5 mm. Dependence of the constants of exponential law on temperature.

Key: (1). kg/mm². (2). s^{-1}.

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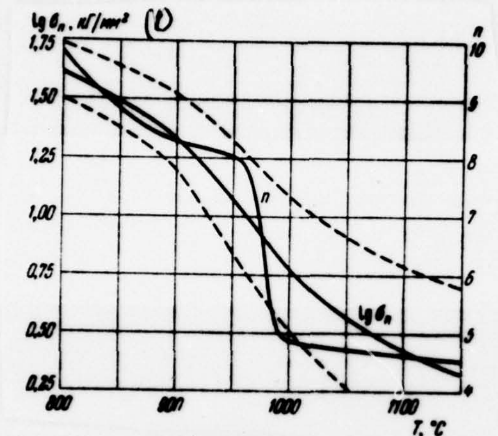

Fig. 127.

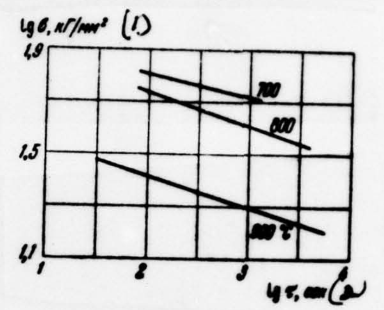

Fig. 128.

Fig. 127. EP-202, sheet 2.5 mm. Dependence cf the constants of power law on temperature.

Key: (1). kg/mm².

Fig. 128. EP-202, sheet 2.5 mm. Dependence cf the time of destruction cn voltage/stress.

Key: (1). kg/mm². (2). s.

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Conditions/mode of the heat treatment: 1) heating to temperature of 1190°C, the holding cf 2 hours, cooling in air, 2) heating to

temperature of 1050°C, the holding of 2 hours, furnace cooling. Hardness after heat treatment HV30=277-279.

Possible approximations of the temperature dependences of the parameters of exponential law in the range of temperatures of 800-1016°C:

$$\varepsilon_e = \varepsilon_{e1,2} \exp \nu T, \quad \sigma_e = \frac{T - T_0}{\beta},$$
$$\varepsilon_{e1} = 8{,}32 \cdot 10^{-18}\ \text{сек}^{-1}\ (1), \quad T_0 = 1090^\circ \text{C},$$
$$\nu_1 = 0{,}0260\ \text{град}^{-1}\ (2), \quad \beta = 57{,}8\ \text{мм}^2\text{град/кГ}\ (3).$$

Key: (1). s^{-1}. (2). deg^{-1}. (3). mm^2deg/kgf.

In the range of temperatures of 1016-1150°C:

$$\varepsilon_{e2} = 1{,}41 \cdot 10^{-7}\ \text{сек}^{-1}\ (1), \quad T_0 = 1285^\circ \text{C},$$
$$\nu_2 = 0{,}00245\ \text{град}^{-1}\ (2), \quad \beta = 216\ \text{мм}^2\text{град/кГ}\ (3).$$

Key: (1). s^{-1}. (2). deg^{-1}. (3). mm^2deg/kgf.

Copper M1 (Fig. 129-131).

The characteristics of rapid creep are determined in the specimen/samples, cut out of their sheet 3 mm in direction of rolling.

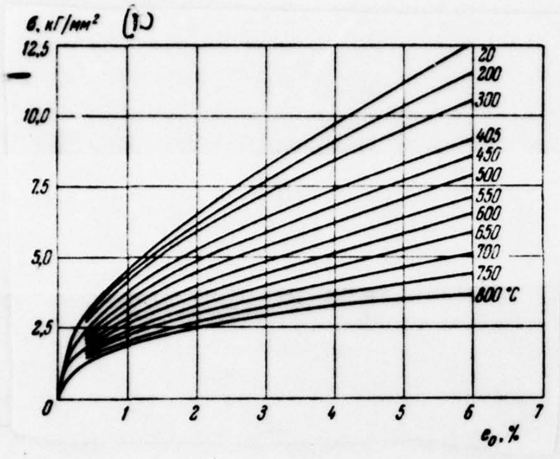

Fig. 129. Copper M1, sheet 3 mm. Curves of instantaneous deformation.

Key: (1). kg/mm².

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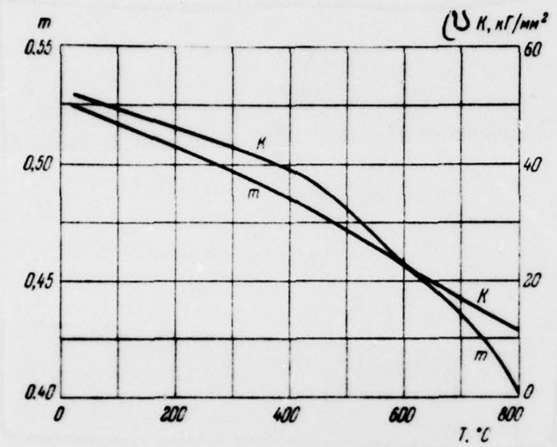

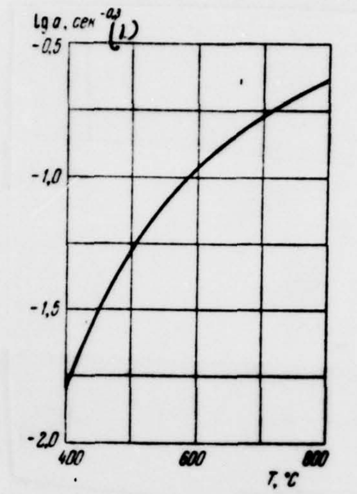

Fig. 130. Copper M1, sheet 3 mm. Dependence on the temperature of constants in the exponential approximation of the curves of instantaneous deformation $\sigma = Ke_0^m$.

Key: (1). kg/mm².

Fig. 131. Copper M1, sheet 3 mm. Dependence on the temperature of value a in the equation of the family of isochronal curves $\sigma = \phi(e_0)/1 + at^{0,3}$.

Key: (1). $s^{-0.3}$.

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Conditions/mode of the annealing: heating in vacuum $5 \cdot 10^{-3}$-10^{-3} tor during 4 hours of 20 min. to temperature of 800°C, holding at this temperature of 2 min. and furnace cooling down to 150°C during 4 hour.

Rapid creep of copper - creep with strengthening. Equation of the family of the isochronal curves

$$\sigma = \frac{\varphi(e_0)}{1 + at^{\beta}}.$$

For all temperatures it is possible to assume β=0.3. Dependence lg a(T) is given in Fig. 131.

The curves of instantaneous deformation ϕ(e) are approximated by exponential function Ke_0^m, the dependence of parameters of which on temperature is given in Fig. 130.

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Addition.

INSTALLATIONS FOR DETERMINING THE CHARACTERISTICS OF RAPID CREEP.

By the authors were utilized two types of testing machines for determining of the characteristics of structural materials under conditions of the rapid creep:

1) lever/crank type installation with the system of loading, ensuring the rapid overload of specimen/sample in accordance with a program of the type of Fig. 94;

2) electromagnetic type installation with the programming of load.

Both installations provide for the rapid heating of specimen/sample by the silit heaters, placed into cylindrical water-cooled reflectors. Temperature controller - on the base of a tool of the type EPP. In the anode circuit of the output tubes of amplifier, is included the winding by the relay which through the intermediate, more powerful relay controls the contactors, which

establish/install the upper and lower limits of stress on silit heaters. The heating rate during the appropriate selection of the power of heater can be reached approximately 25-30 deg/s during heating to 800-1000°C. Oscillation/vibrations of temperature in time - order 1o/o of the supported level. Along the length of working part, the temperature is levelled off by changing the position of heaters relative to specimen/sample. In certain cases it is necessary to establish/install the shields, which overshade the center section of the specimen/sample.

During installations are utilized the identical in form and size/dimensions specimen/samples whose drawings are given to Fig. 132 and 133. Section/cut of specimen/sample out of the working zone (60 mm), limited by rectangular shoulder/fillets, the same (at length 110 mm)as the section/cut of working part.

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This does not threaten with the rupture of specimen/sample out of working part, since the temperature of these sections is lower than working section. But if this rupture occurs, then this signals about the disturbance of the normal temperature field of specimen/sample.

Strain measurement of specimen/sample is manufactured by extensometer with elastic sensing element whose strain is measured by the wire-strain gauges.

Recording strains is manufactured either with the aid of loop cscillograph and the extenscmetric amplifier (this method is convenient for comparatively accelerated tests), or with the aid of an electronic potentiometer of the type of EPP with somewhat by a changed bridge circuit (this method is convenient for sufficiently long-terms test).

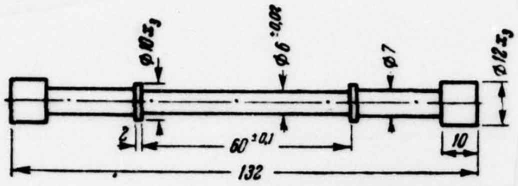

Fig. 132.

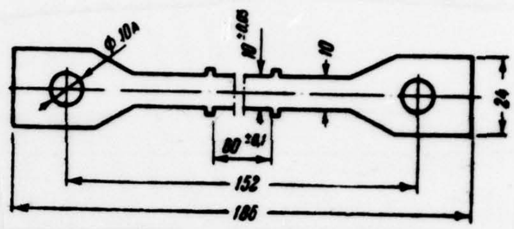

Fig. 133.

Fig. 132. Circular specimen/sample.

Fig. 133. Flat/plane specimen/sample.

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The sensitivity of the measuring circuit of strain can be changed, by changing the factor of amplification of extensometric amplifier or the feed current of the bridge of EPP.

Possible limits of a change in the sensitivity of 0.5-50 μm/mm. Accuracy/precision of measurement 5 μm. Linear section 1-5 mm.

Load on specimen/sample during lever/crank type installation is applied with the aid of suspension by 6 and the system of magnets 0m - 5m (Fig. 134). Loads 1-5 either lie/rest on the cross-beams of suspension, and then their weight is applied to specimen/sample or

are attracted to the appropriate magnets 1m - 5m. The weight of the i cargo is equal to $q \cdot 2^{i-1}$ (q - led to specimen/sample weight of cargo 1). Leaving during suspensicn different combinations of loads, it is possible to obtain with interval in q any lcad from q to 31 q.

With overload first is included magnet 0m, which attract/tightens armature 7, which is rigidly connected with the plunger of damper 8, attached on mounting. Moving upward, armature through brackets 9 arrests framework by 6. The clearance between the loads, which lie on suspension, and by the end/faces of the corresponding magnets is reduced tc 0.5-1 mm, the same clearance remains between the lower end/faces of the loads, attracted to magnets, and the corresponding cross-beams cf framework 6. This creates conditions for reliable capture by the magnets of loads and smooth lowering of loads for suspension. After the regrouping of loads, the magnet 0m will temper armature 7, which smoothly disengage suspension. The new combination of loads is prcved to be applied to specimen/sample.

The simple electromechanical servo system controls in this case the operating position cf lever. The electrical circuit of load dispatching provides the automatically necessary sequence of operations in time.

In second type installaticn (Fig. 135) the load to specimen/sample is applied with the aid of the electromagnet whose coil is fed by amplidyne EMC. The error signal, equal to a difference in the lumped voltage and exit stress of the extensometric amplifier at input of which are irclucled the strain gauges, stuck on dynamcmeter, enters the amplifier UE-109.

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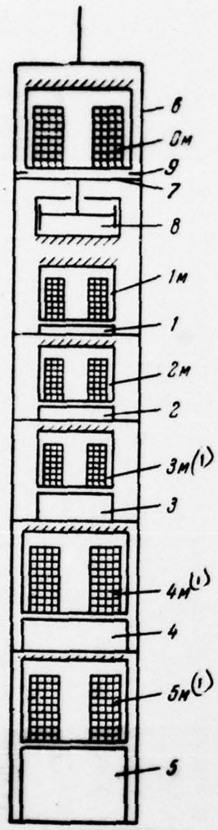

Fig. 134.

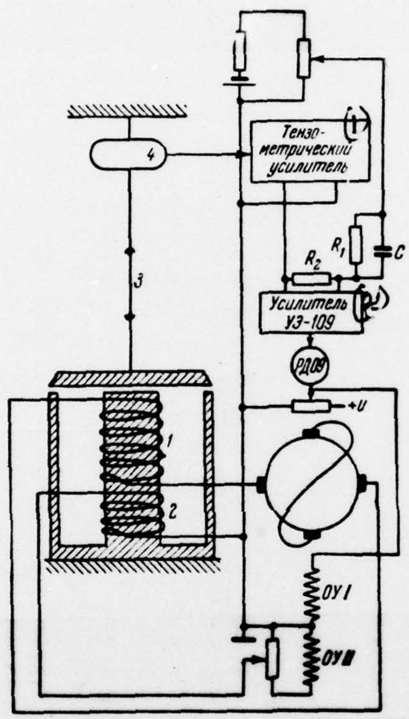

Fig. 135.

Fig. 134. Assembly of loading of lever/crank type installation.

Key: (1). m.

Fig. 135. Common/general/total block diagram of installation with electromagnetic loading. 1 - power coil, 2 - feedback coil, 3 - specimen/sample, 4 - dynamometer.

Key: (1). Extensometric amplifier. (2). Amplifier UE-109.

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Engine RD-09, controlled by this amplifier, establish/installs, rotating the sliding contact of potentiometer, which corresponds coil current of control of EMU. The second control winding is utilized for the negative feedback, voltage/stress for which is taken from the coil, inductively connected with power magnet coil. In summation, the force varies in proportion to tc lumped voltage which it is possible to program sufficiently arbitrarily. With the rapid elongation, necessary for recording the curves of instantaneous deformation, the feedback can be establish/installed positive. In this case the time of elongation can be led to 0.1 s.

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